Introduction to the theory of confinement

X. Garbet

CEA Cadarache

Confinement is a crucial issue for fusion

- Lawson criterion for ignition

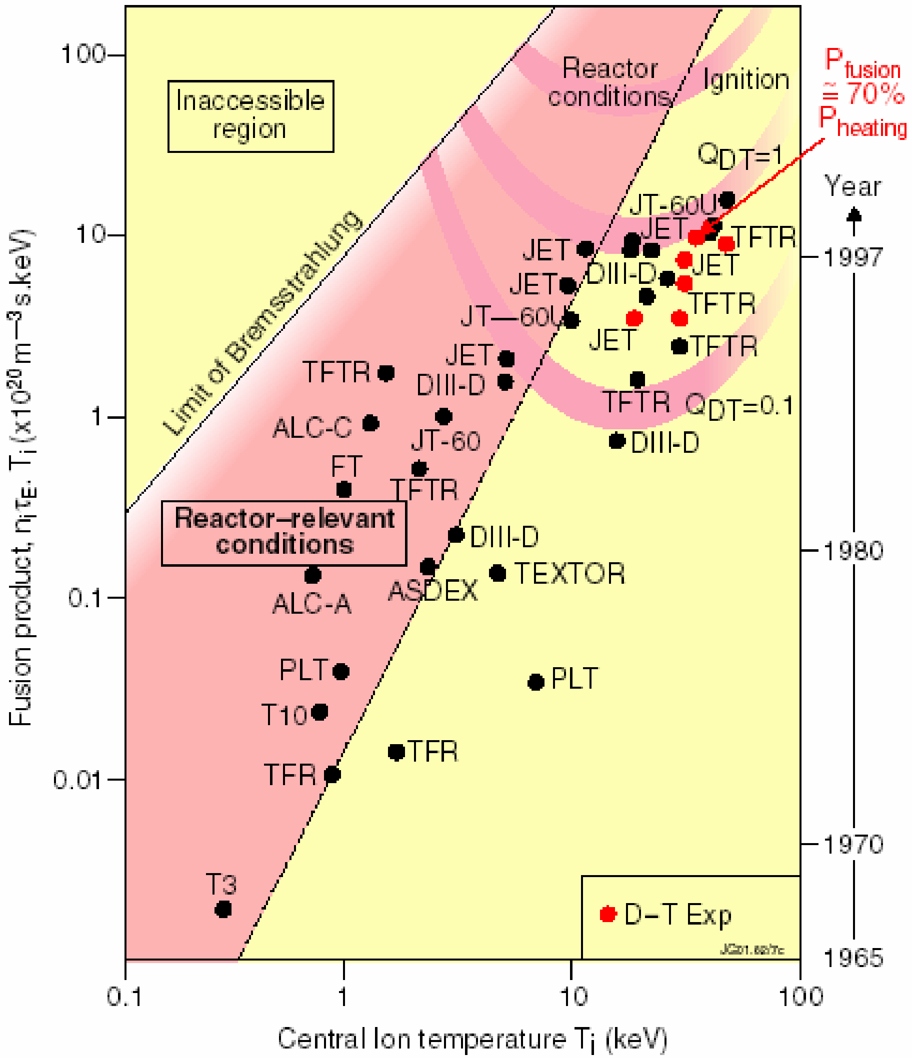
$n_D T_D \tau_E = 3\ 10^{21} m^{-3}.keV.s$

- Confinement

$$\tau_E = \frac{\text{Energy content}}{\text{Power losses}}$$

~3.7 s in ITER

$\rightarrow$ Transport



Some orders of magnitude

- Heat flux equation

$$\frac{3}{2} n \partial_t T + \nabla \cdot \phi_T = S$$

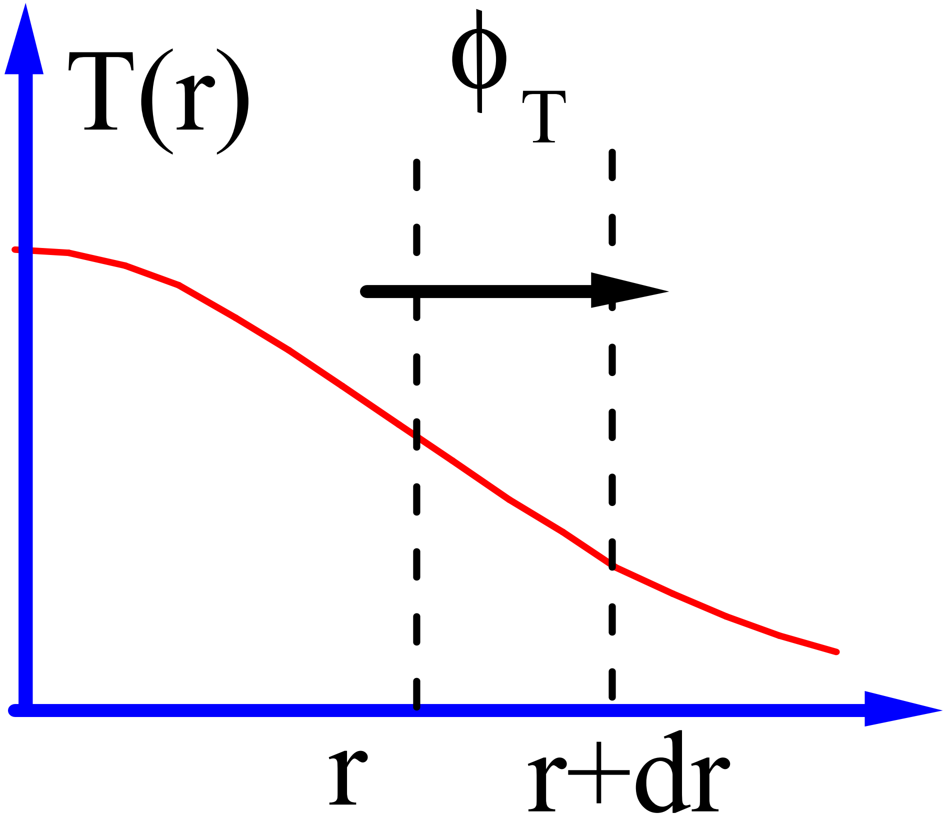
$$\phi_T = -n \chi_T \nabla T$$

- Transport in a tokamak is diffusive

$$\tau_E \approx \frac{a^2}{\chi_T}$$

- If $\tau_E \approx 1\text{s}$ and $a \approx 1\text{m}$, then

$$\chi_T \approx 1\text{m}^2\text{s}^{-1}$$



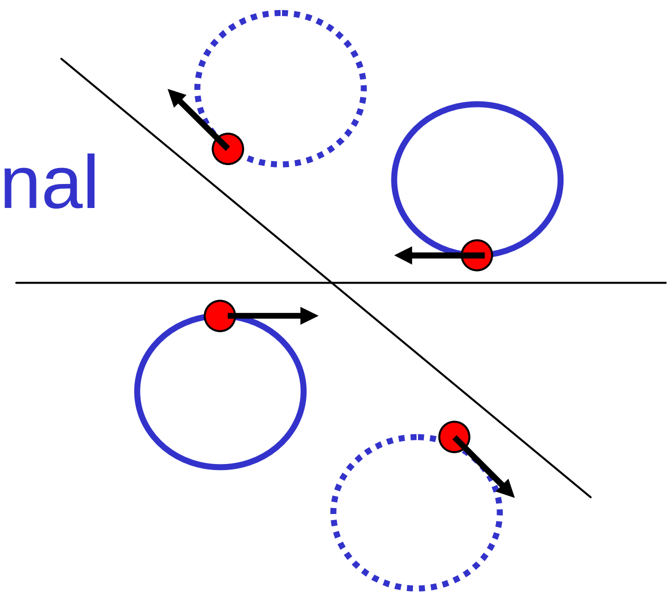
Orders of magnitude (cont.)

- Collisional transport :

random displacement ≈ ρ_c

every collisional time $1/\nu_c$

$\rightarrow \chi_{T,coll} \approx \nu_c \rho_c^2$



- Neoclassical theory: enhanced collisional transport due to magnetic pumping

$\rightarrow \chi_{T,neo} \approx q^2 /(r/R)^{3/2} \nu_c \rho_c^2$

Ions $\chi_{Tcoll,i} \approx 0.1 m^2 s^{-1}$

Electrons $\chi_{Tcoll,e} \approx 0.001 m^2 s^{-1}$

- Usually smaller than experimental value.

- Losses are mainly conductive

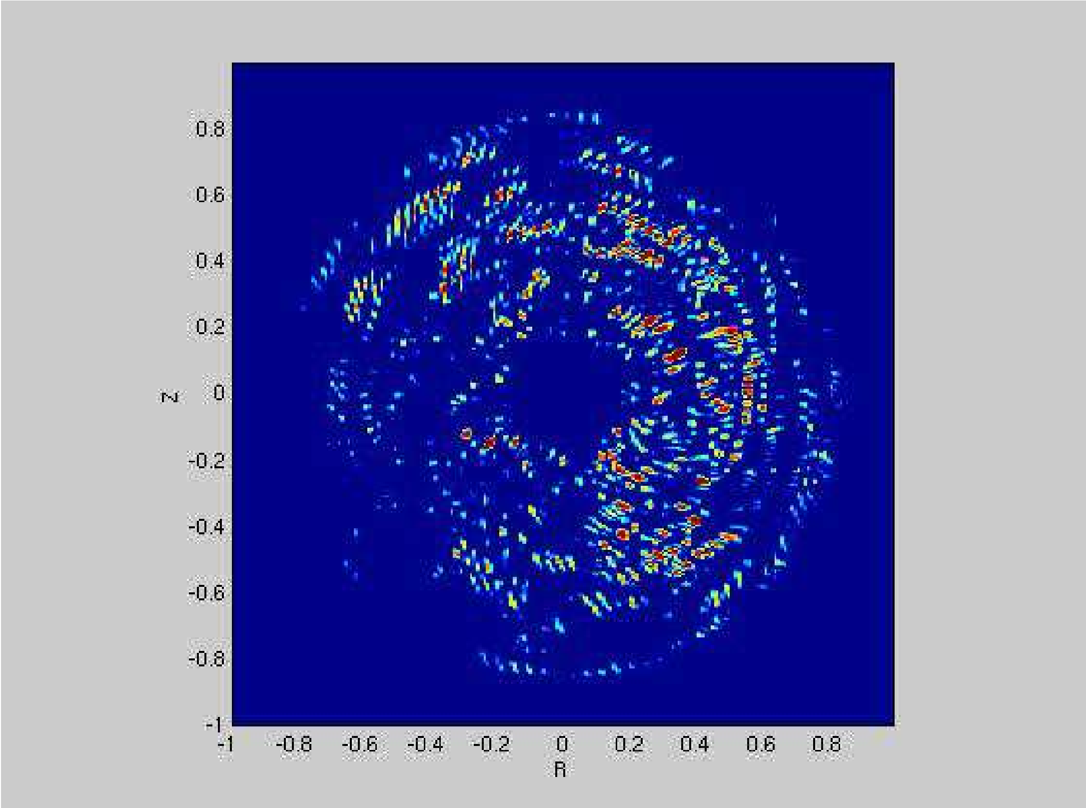
$$\tau_E \approx \frac{a^2}{\chi_{turb}}$$

→Turbulent diffusion χ_{turb} determines the confinement.

- However:

- parallel transport is nearly collisional,
- collisional transport can be dominant in transport barriers.

Turbulent transport is dominant



Turbulent flux

Outline

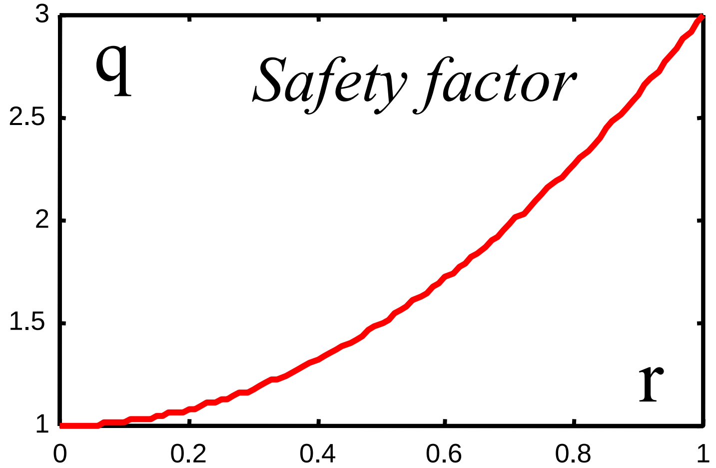
1) Basics of turbulent transport: turbulent diffusion, instabilities, turbulence modelling.
2) A powerful approach: dimensionless analysis.
3) Status of our understanding of turbulent transport: heat, particle, momentum.
4) Building a transport model: mixing-length estimate, quasi-linear theory.
5) Why is predicting difficult? Turbulence self-organization.
6) Improved confinement, physics of transport barriers.

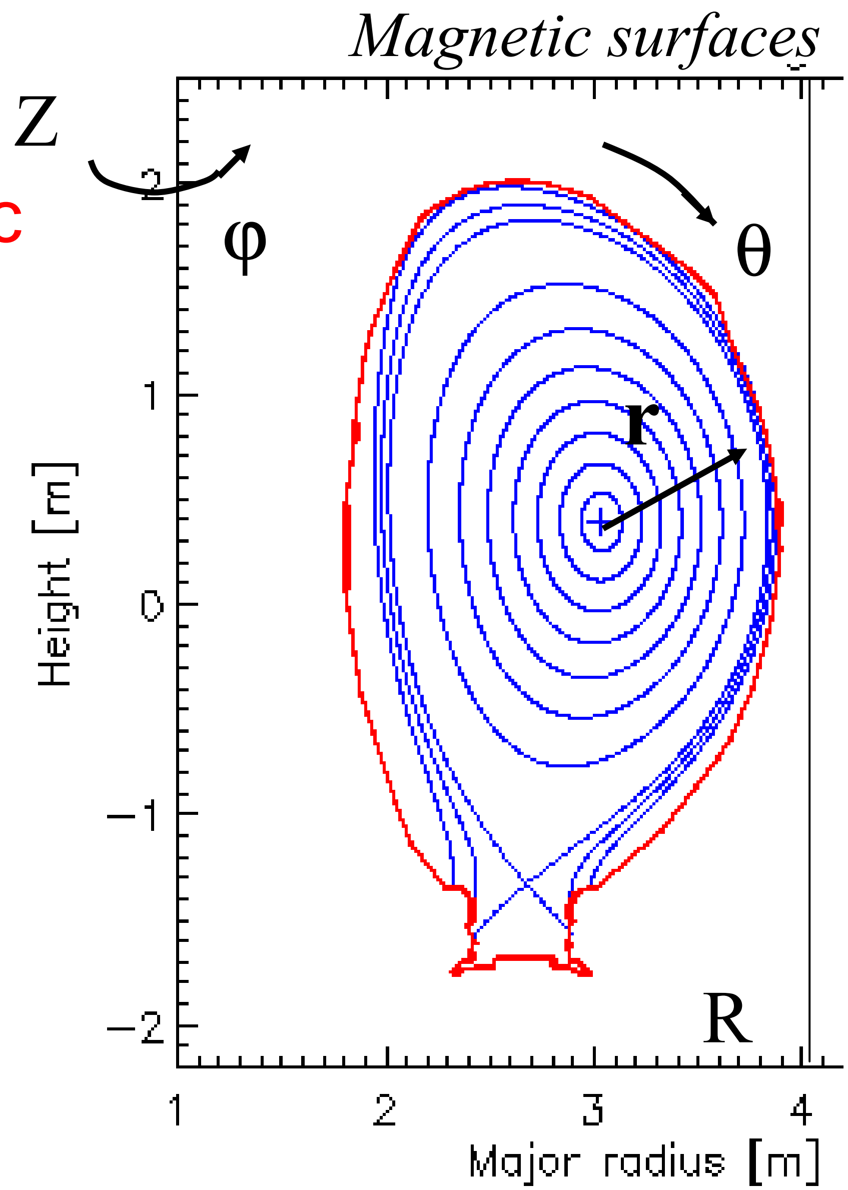
Part I - Basics

- A few reminders.
- Basics of turbulent transport: random walk, main instabilities.
- Some key ingredients of theory and modelling.

Geometry

- Field lines generate magnetic surfaces.
- Safety factor : $q(r) = \dfrac{d\varphi}{d\theta}$
- Density and temperature are constant on magnetic surfaces.

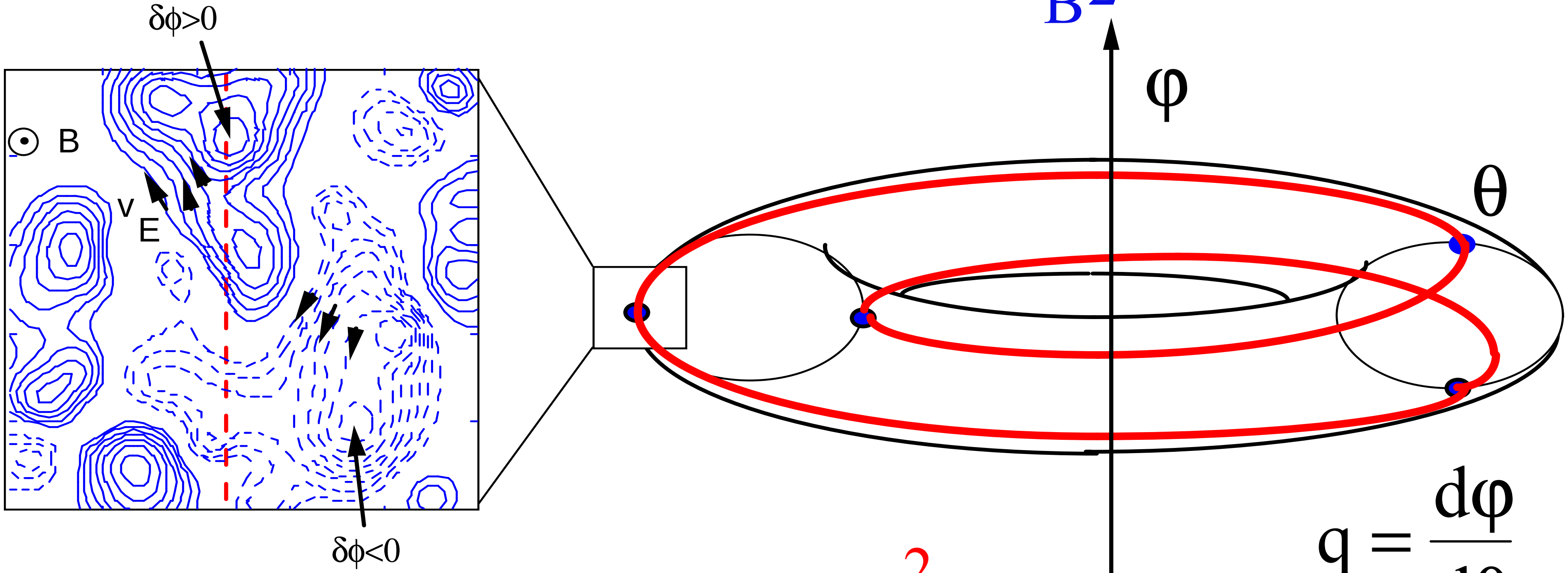




Fluctuations of ExB drift velocity produce turbulent transport

E×B drift velocity

$$v_E = \frac{B\times\nabla\phi}{B^2}$$



$$q = \frac{d\varphi}{d\theta}$$

$$D_{turb.} \approx \left|v_E\right|^2 \tau_c$$

Random walk process

- ExB drift

$$v_E = \frac{B \times \nabla\phi}{B^2}$$

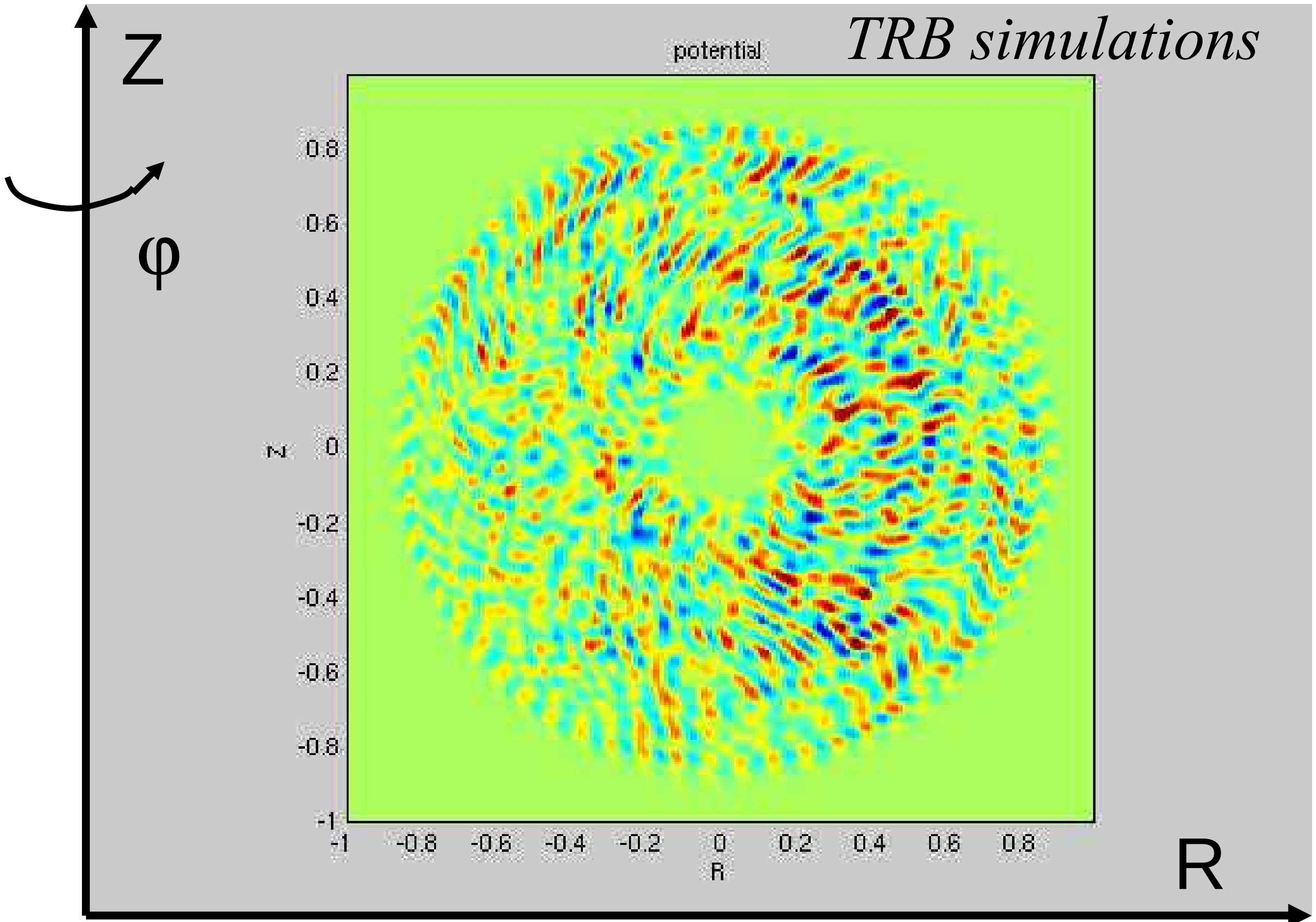
- Turbulent diffusion

$$D_{turb} \propto |v_E|^2 \tau_c$$

$$\propto L_c^2 / \tau_c$$

- Turbulent flux

$$\phi_E = \frac{3}{2}\langle p v_E \rangle$$



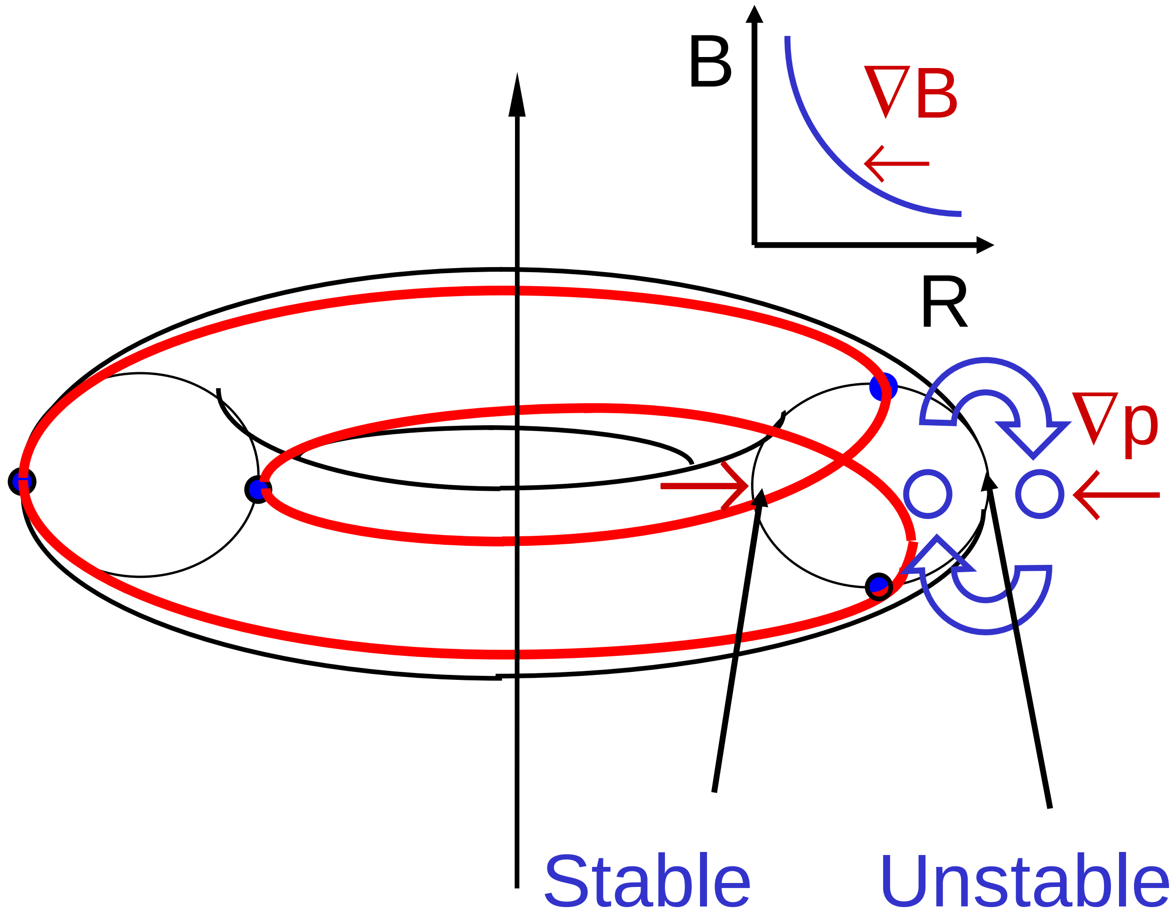
Contour lines of electric potential ϕ.

Main instabilities are interchange modes

- Exchange of two flux tubes is energetically favourable if

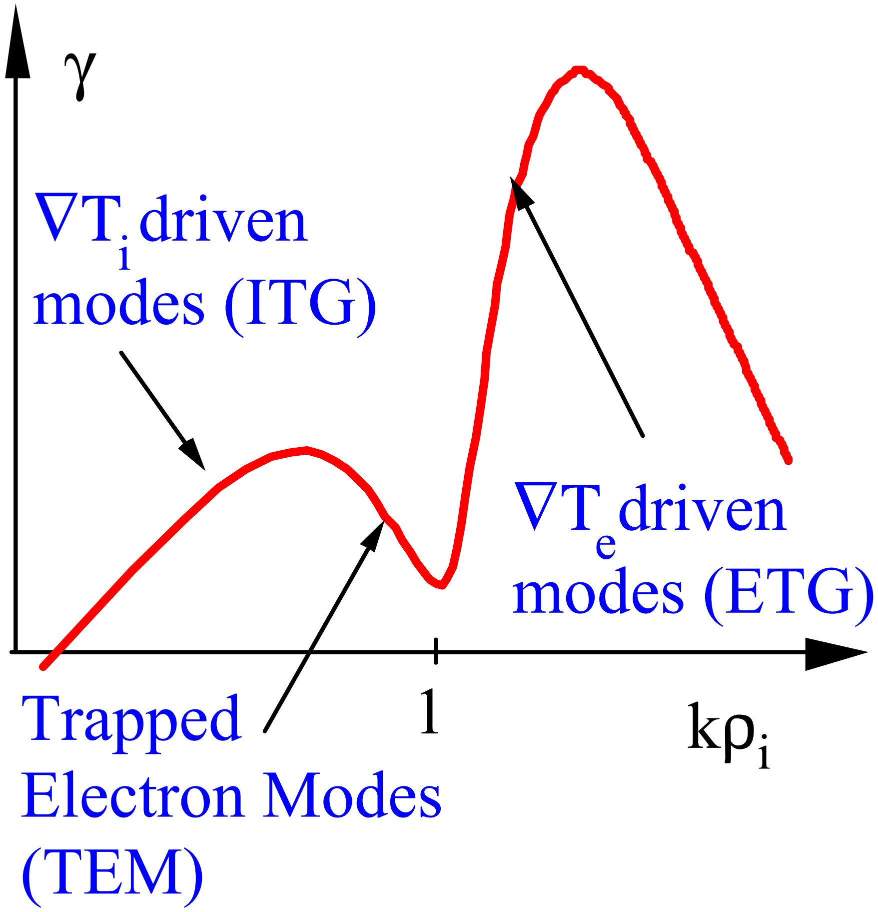
$(v_E \cdot \nabla B)(v_E \cdot \nabla p) > 0$

- Stable and unstable regions are connected by field lines.



Several branches are potentially unstable

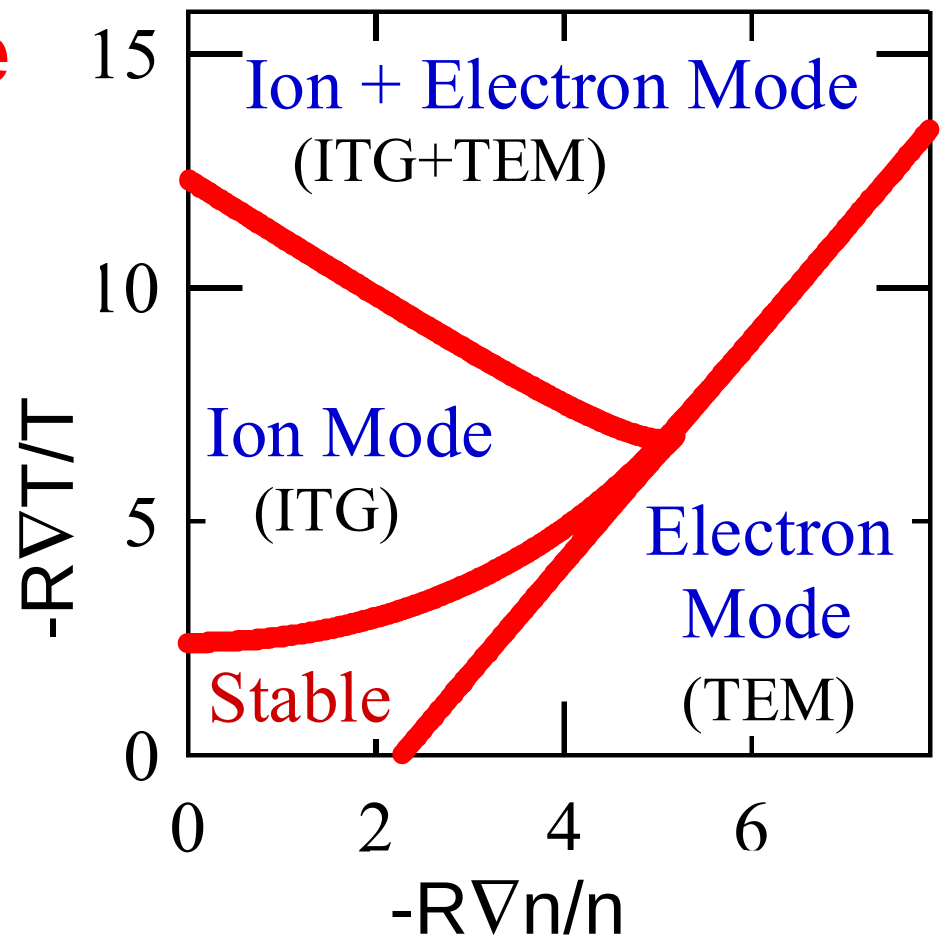
- Ion Temperature Gradient modes: driven by passing ions, interchange + “ slab ”
- Trapped Electron Modes: driven by trapped electrons, interchange type.
- Electron Temperature Gradient modes: driven by passing electrons
- Ballooning modes at high β



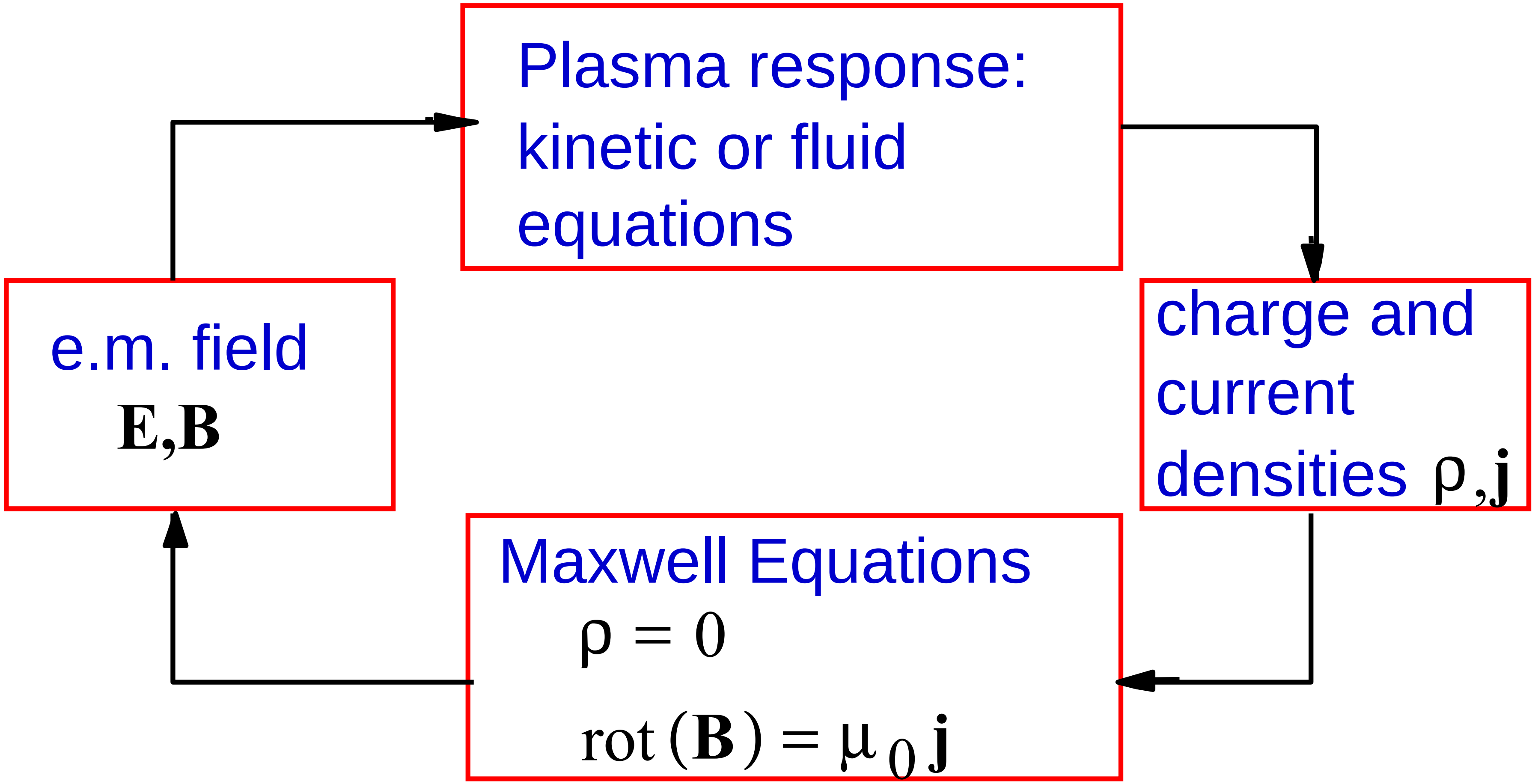
Electron and/or ion modes are unstable above a threshold

- Instabilities → turbulent transport
- Appear above a threshold κ_c.
- Underlie particle, electron and ion heat transport : interplay between all channels.

Stability diagram -Weiland model



A Self-Consistent Problem



Calculating the plasma response: fluid equations

Continuity equation

$$d_t n = -n\nabla.\mathbf{V}$$

Force balance equation

$$nmd_t\mathbf{V} = -\nabla p - \nabla.\boldsymbol{\pi} + ne(\mathbf{E} + \mathbf{V}\times\mathbf{B})$$

Heat equation

$$d_t p = -5/3\, p\nabla.\mathbf{V} - 2/3\nabla.\mathbf{q} - 2/3\boldsymbol{\pi}:\nabla\mathbf{V}$$

Lagrangian derivative $d_t=\partial_t + \mathbf{V}.\nabla$

No wave particle resonant interaction, nor orbit effects: partly cured with closure schemes and gyroaverage operators.

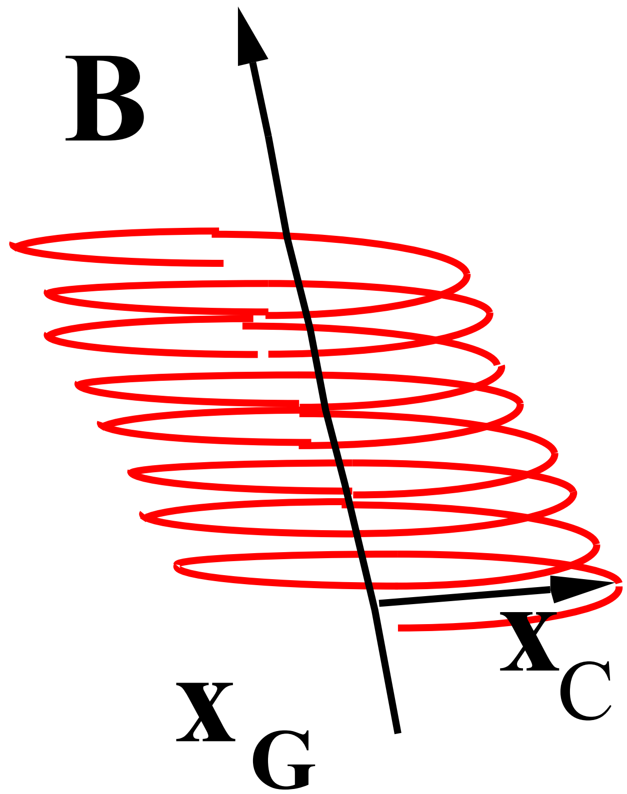
Gyrokinetic theory

- Kinetic equation

$$\partial_t F + \underbrace{\dot{\mathbf{x}}\partial_{\mathbf{x}} F + \dot{\mathbf{p}}\partial_{\mathbf{p}} F}_{= -[H,F]} = 0$$

- In principle a 6D calculation!
- However $\omega_{turb} << \Omega_c$

$\rightarrow \quad \mu = \dfrac{m_i v_\perp^2}{2B(\mathbf{x_G})}$ is an invariant



Gyrokinetic theory

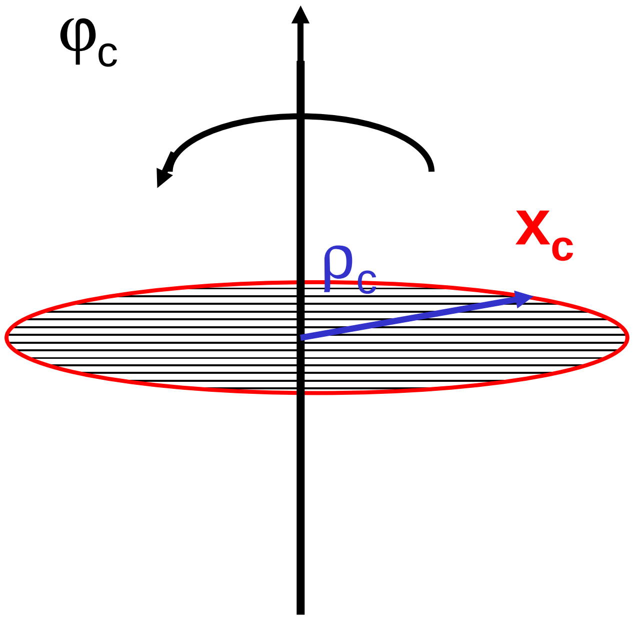
Brizard and Hahm 08

- Compute the distribution of gyrocenters $\overline{F}$

$$\partial_t \overline{F} - [\overline{H}, \overline{F}] = 0$$

$$F = B^{-1} \partial_\mu F_{eq} \left(H - \overline{H} \right) + \overline{F}$$

- $\overline{H}$ is the hamiltonian averaged over the fast motion (gyroaverage).

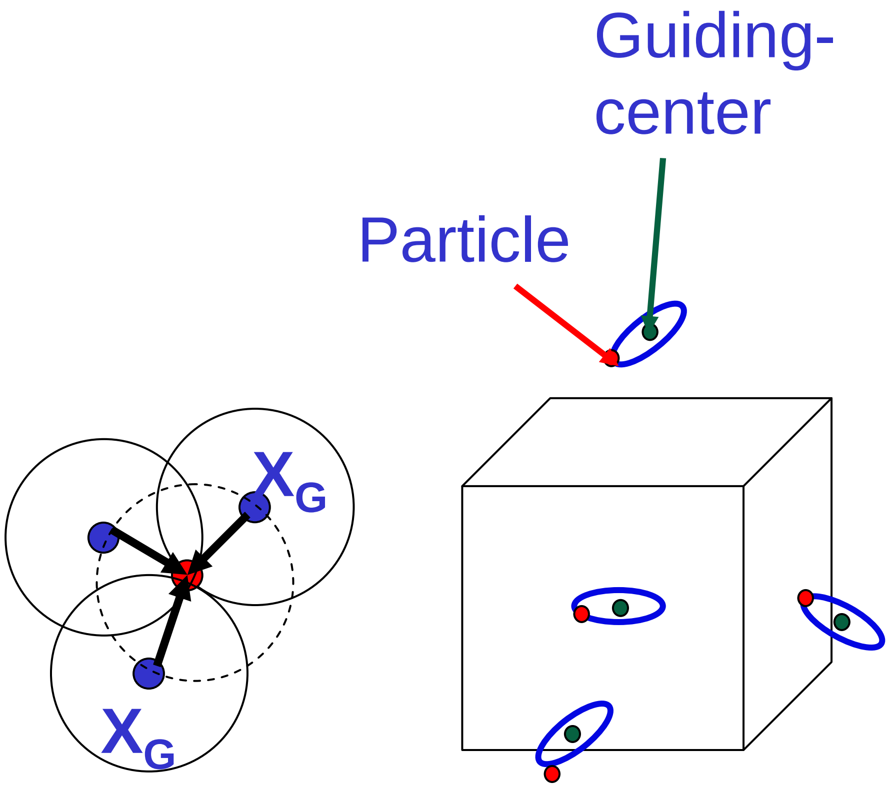


Coherence in gyrokinetics

- Maxwell equations: local charge and current densities

- Must be related to gyrocenter charge and current densities: an other gyroaverage!

- Difference between F and $\bar{F}$ is the polarisation term.



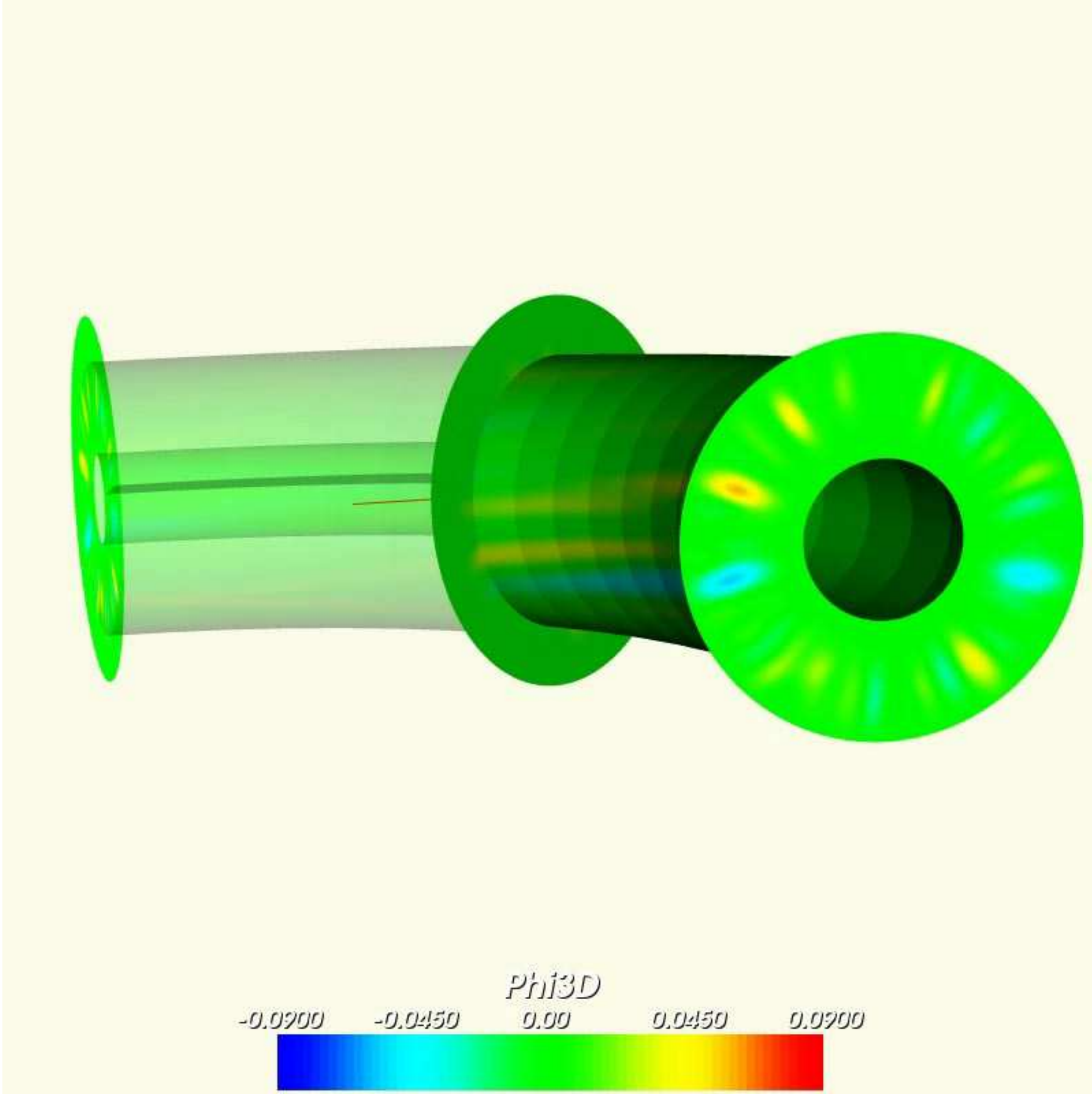
$$n(\mathbf{x},t) = \int \frac{d\varphi_c}{2\pi} \int d^3\mathbf{p}\,\bar{F}\left(\mathbf{x}-\mathbf{x_c},\mathbf{p},t\right)$$

After a lot of work to develop a gyrokinetic code (see lecture by C.S. Chang) …

Numerical simulations reproduce the main expected features of turbulence

-Structures aligned with the magnetic field.

- fluctuations are ballooned on the low field side.



Part II - Dimensionless scaling laws

- Similarity principle.
- Numerical and experimental tests.
- Extrapolation using dimensionless scaling laws.

Dimensionless numbers

Kadomtsev '75

- Counting the dimensionless parameters for a given set of plasma parameters
- 8 numbers for a pure e-i plasma

I. $\nu^* = qR/\lambda_{mfp}$ $\rho^* = \rho_c/a$ $\beta = 2\mu_0 p/B^2$

Larmor radius

II. $A = R/a$ $\tau = T_e/T_i$ q

III. $\mu = m_e/m_i$ $N = n_e\lambda_d^3$

- Implications on confinement time, II and III given

$$\omega_c\tau_E = F(\rho^*, \beta, \nu^*)$$

Scale invariance

Connor&Taylor '77

- Analysis of scale invariance of Fokker-Planck equation coupled to Maxwell equations $\rightarrow$ local relations.

- If geometry, profiles, and boundary conditions are fixed, plasma is neutral, then

$$\chi = \frac{T}{eB} G(\rho^*, \beta, \nu^*)$$

Bohm diffusion coefficient

Dimensionless scaling is a powerful tool to predict transport in a next step device

Similarity principle

$$\omega_c \tau_E = F(\rho_*, \beta, \nu_*)$$

Normalised gyroradius:

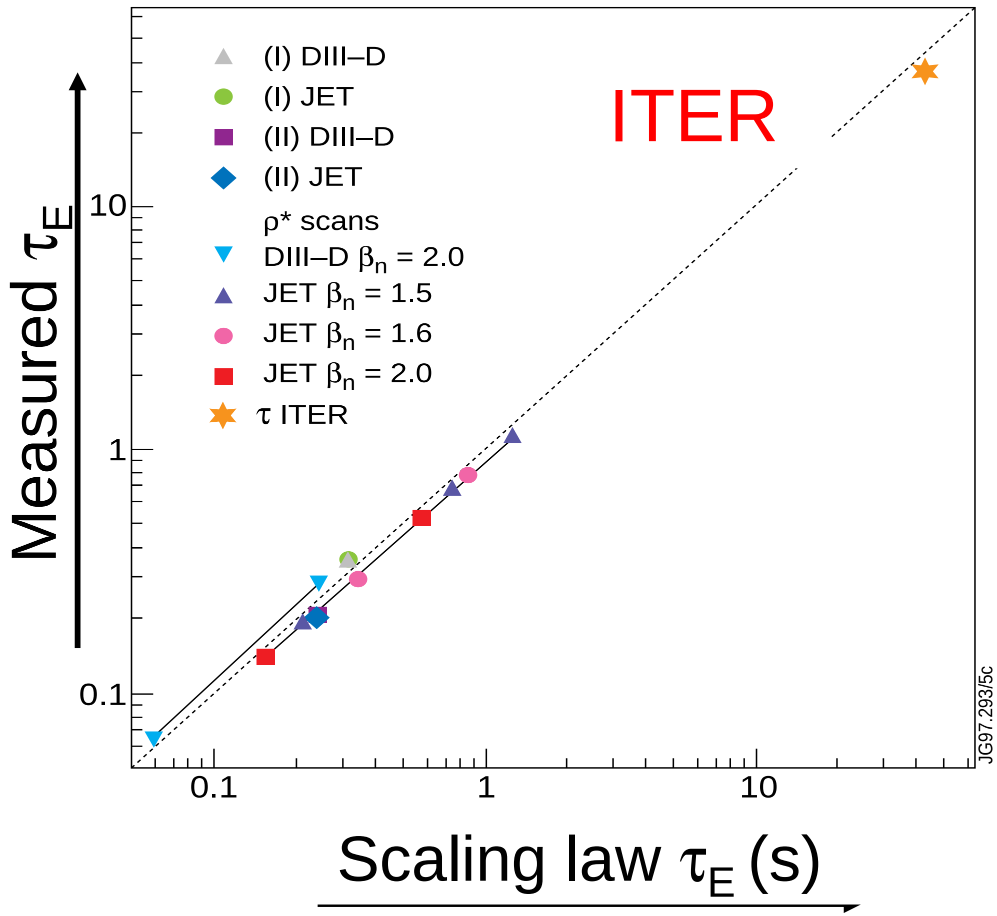
$$\rho_* = \frac{\rho_c}{a}$$

beta: $\beta = \dfrac{p}{B^2 / 2\mu_0}$

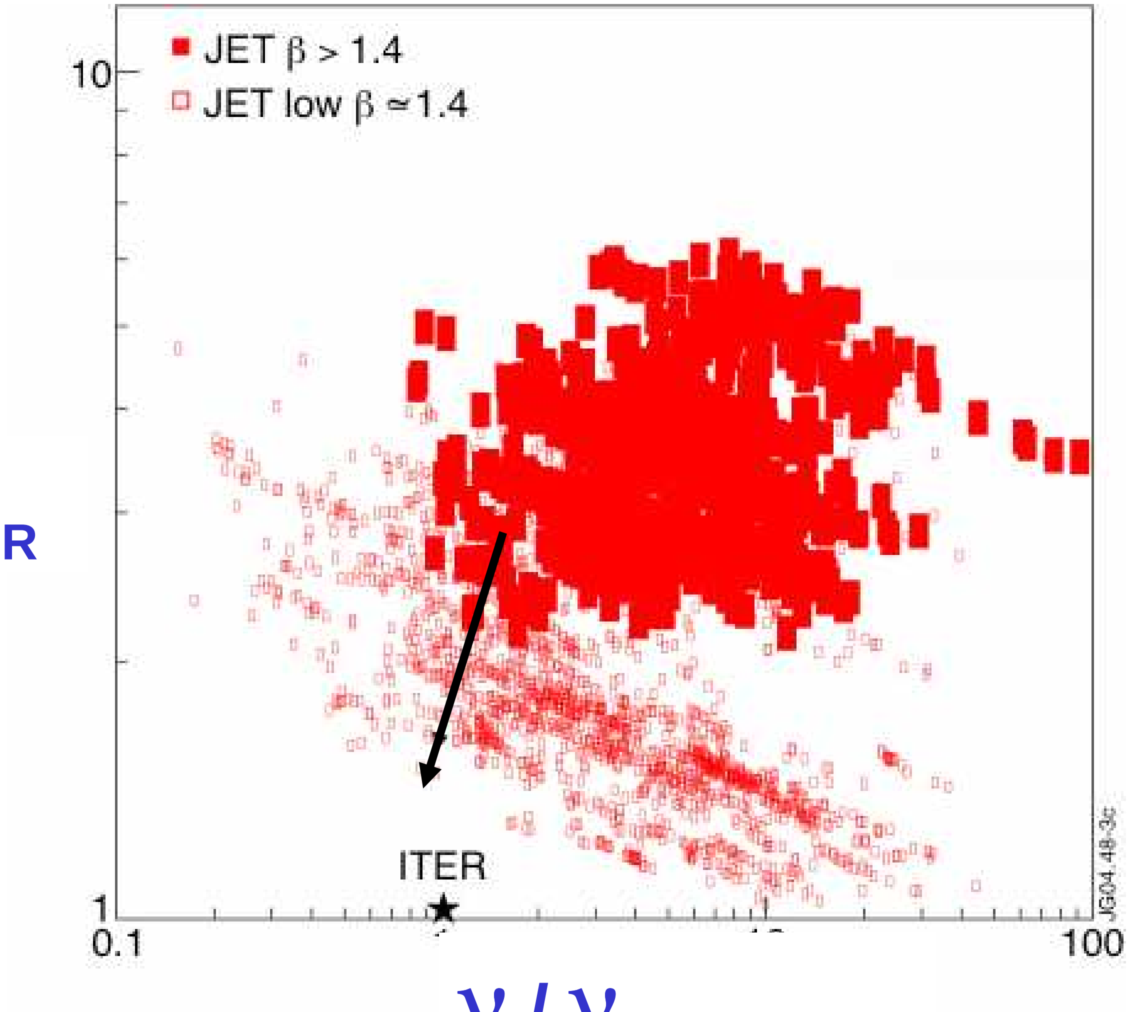
collisionality:

$$\nu_* = \frac{\nu_{coll}}{c_s / R}$$

Measured τ_E vs fit, ITPA



ρ_* and ν_* will be smaller in ITER



ρ_* / ρ_{*ITER}

ν_* / ν_{*ITER}

What is gyroBohm scaling law?

- At fixed β and ν^*,

$$\frac{L_c}{a} \equiv [\rho_*]^{\frac{\alpha+1}{2}} \quad \gamma \equiv \frac{c_s}{a} \rightarrow \quad \chi \equiv \frac{T}{eB}[\rho_*]^{\alpha}$$

- Two main cases: $\alpha=1$ (gyroBohm) and $\alpha=0$ (Bohm).

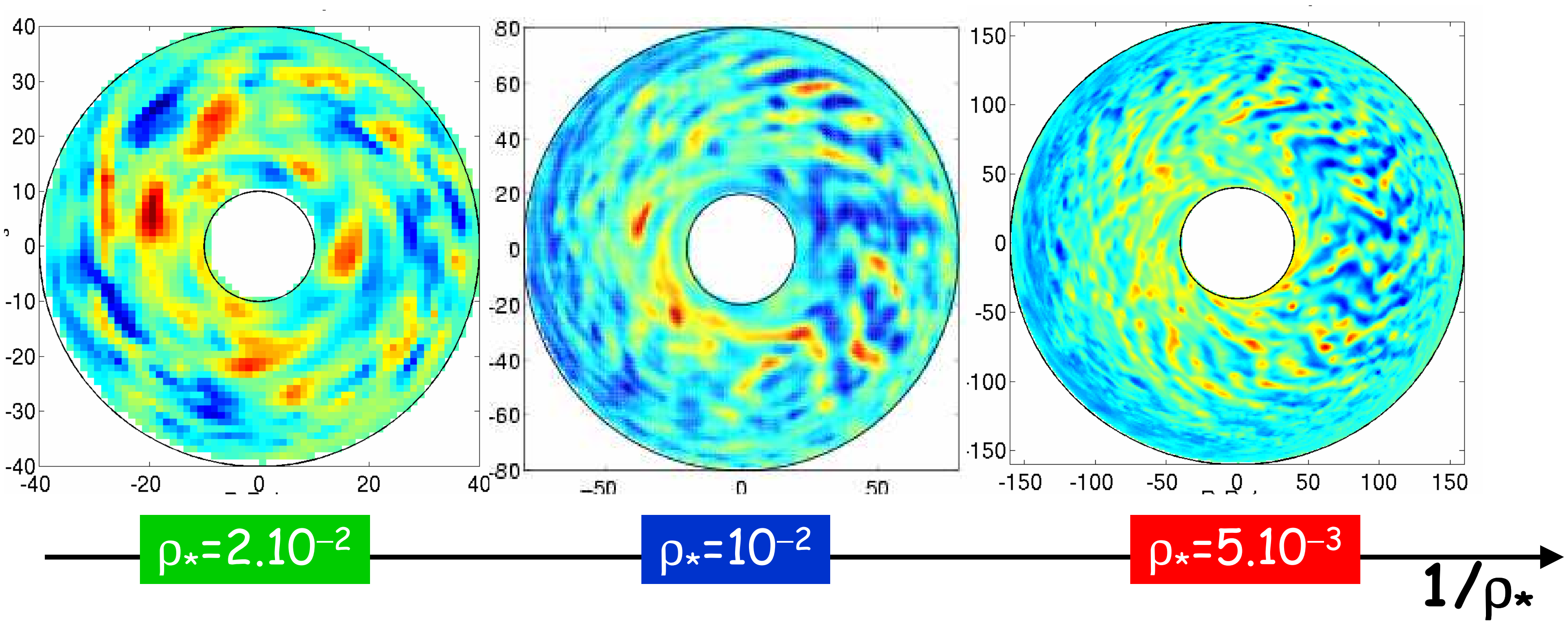
- Theory predicts that when $\rho_* \rightarrow 0$, the scaling is gyroBohm

$$\chi \equiv \frac{T}{eB}\rho_*$$

An example of gyroBohm scaling

- Simulations where the scale ρ* is changed by a factor 4
- Agree with $L_c \equiv \rho_c$ and $\chi \equiv (T/eB)\,\rho_c/a \rightarrow \omega_c \tau_E \equiv \rho_*^{-3} F(\beta, \nu_*)$

Sarazin 07



Scaling law with ρ_* is close to the theoretical expectation

- ITER scaling law

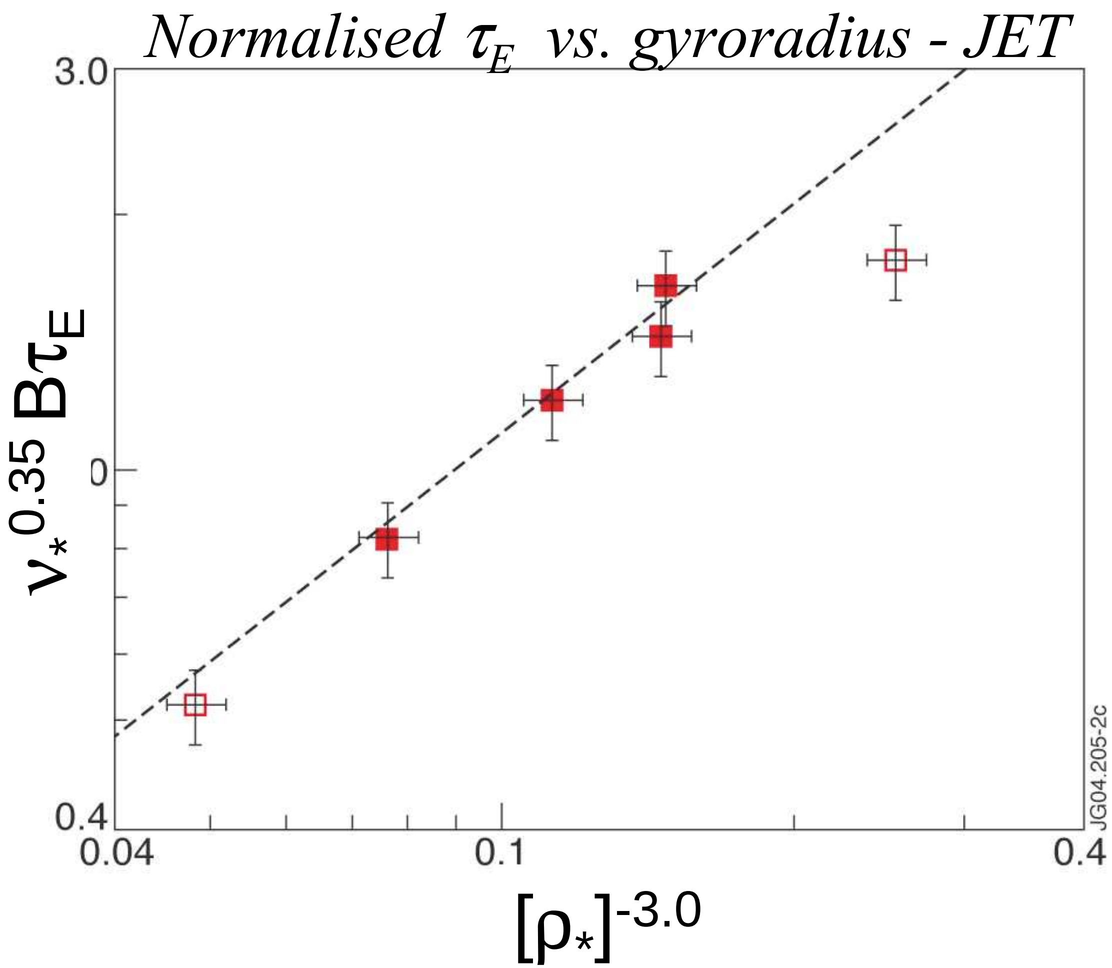
$\omega_c\tau_E \sim \rho_*^{-3.0}\ \beta^{-2.9}\ \nu_*^{0.0}$

- Experiments on DIIID and JET

$\omega_c\tau_E \sim \rho_*^{-3.0}\ \beta^{0.0}\ \nu_*^{-0.35}$

- Consistent with gyroBohm scaling law for electrostatic turbulence:

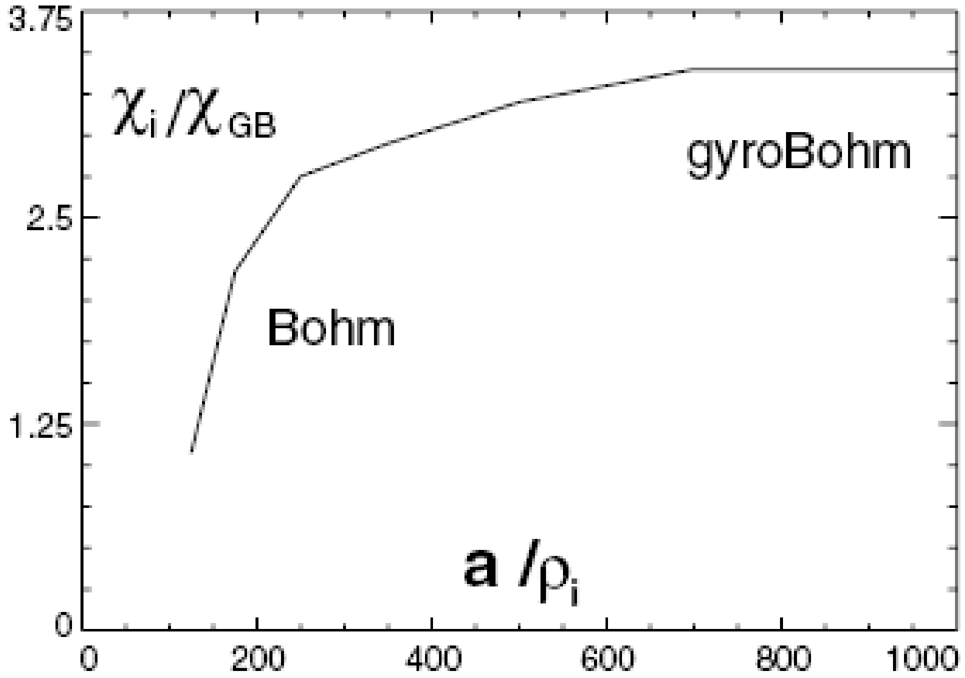
$\omega_c\tau_E \sim \rho_*^{-3.0}\ \beta^{0.0}\ \nu_*^{?}$



Scaling is gyroBohm when $\rho^* \to 0$

- Gyrokinetic and fluid simulations find that the scaling is gyroBohm when $\rho^* \to 0$
- The critical value of ρ^* for Bohm to gyroBohm scaling is still subject to debate.
- Cause for Bohm scaling is controversial.

Lin 02



GyroBohm scaling law is favorable for ITER

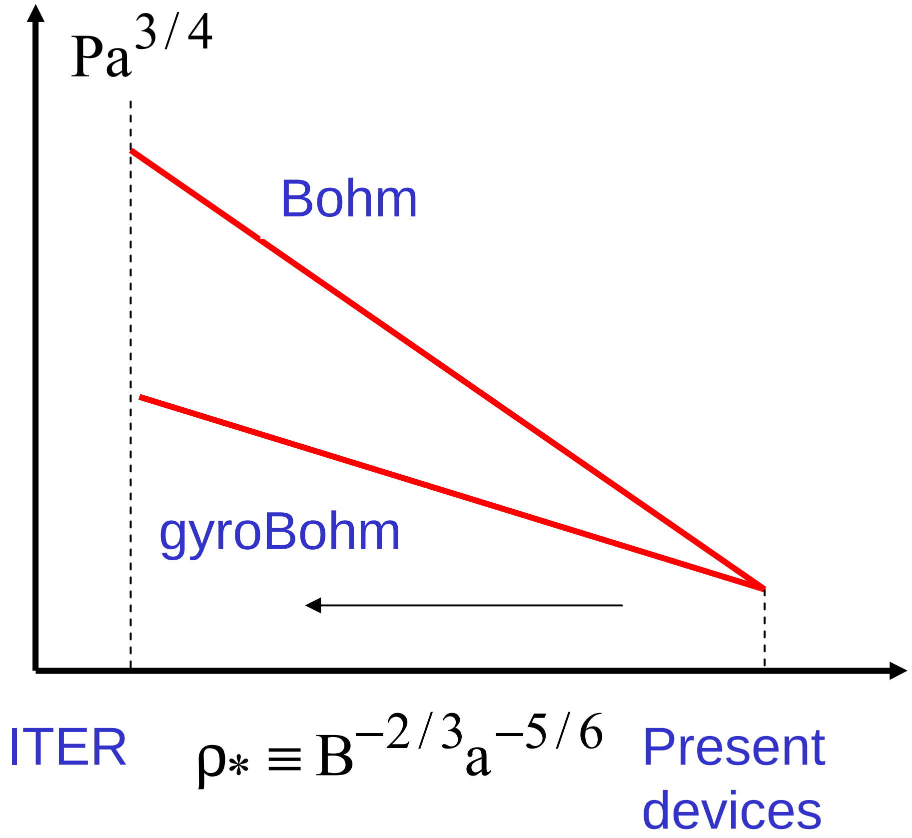
- At constant β and ν^* the normalized loss power $Pa^{3/4}$ is a function of

$$\rho_* \equiv B^{-2/3} a^{-5/6}$$

only, i.e.

$$Pa^{3/4} \equiv [\rho_*]^{\alpha - 5/2}$$

- GyroBohm scaling corresponds to the lowest losses.



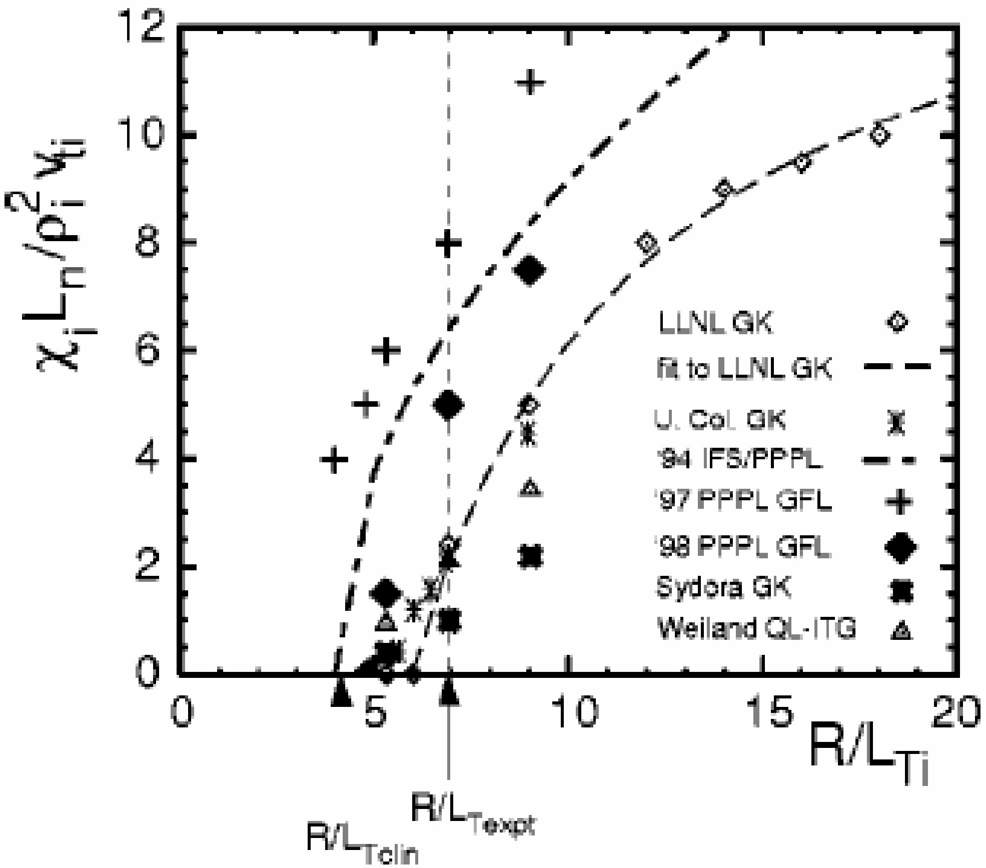
Part III
Status of the understanding for each transport channel

- Ion heat transport
- Electron heat transport
- Particle transport
- Momentum transport

Ion heat transport is rather well understood

- ITG dominated: quite well assessed.
- Has become a test for gyrokinetic codes
- Still some issues: turbulence spreading, Dimits shift, etc…

Dimits 00



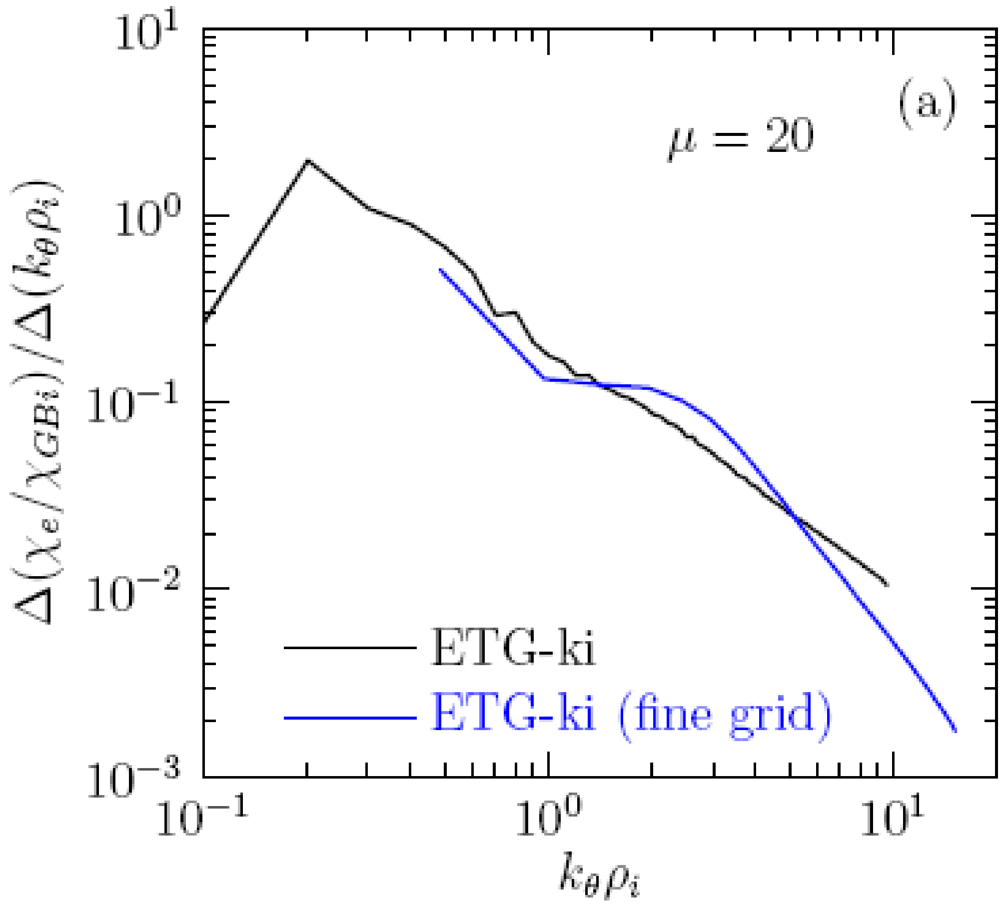
Electron heat transport

- Large contribution from TEMs
- Contribution from ETGs still a debated issue:

\- small for ITG dominated turbulence Candy 06

\- might be significant for TEM/ETG dominant modes Jenko 08

Candy 06



Particle transport

- Particle flux

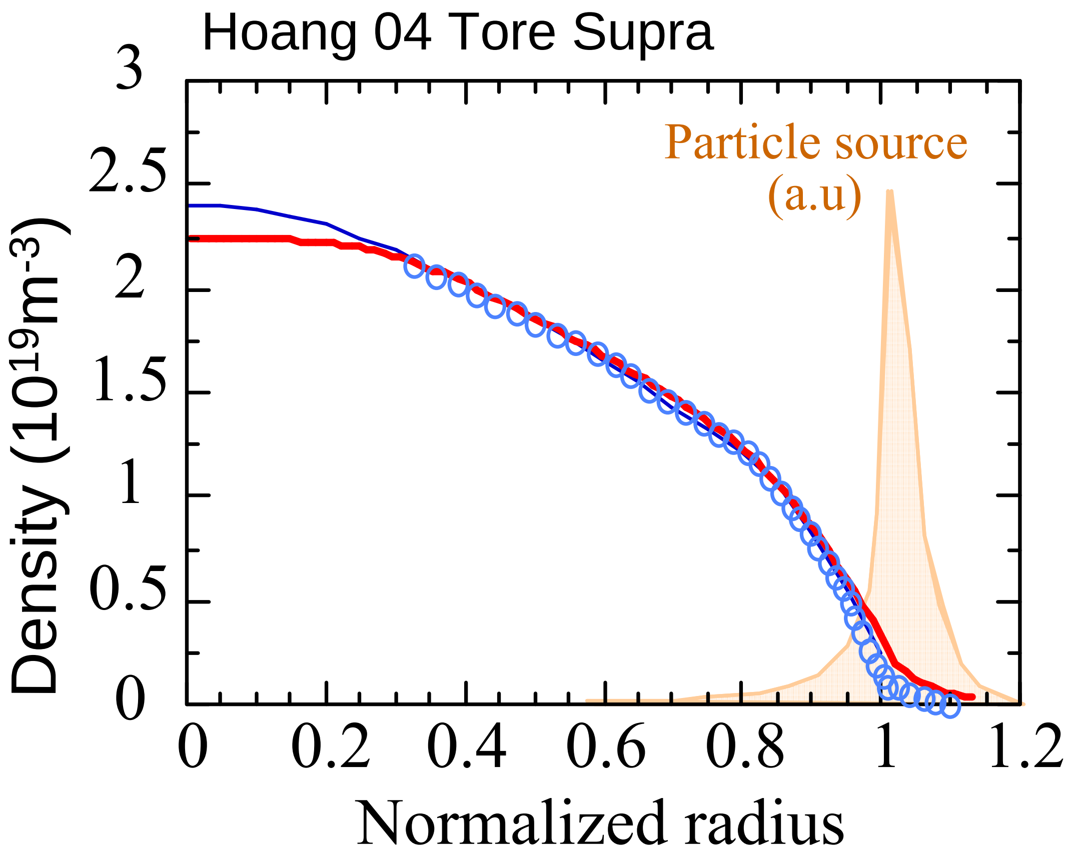
$$\Gamma_e = -D\frac{dn_e}{dr} + Vn_e$$

- Diffusion is turbulent

$$D = D_{turb}$$

- pinch velocity= collisions + turbulence

$$V = V_{neo} + V_{turb}$$



- In a reactor:

- ionisation source localised in the edge $\rightarrow \Gamma_e=0$

-V_{neo} ~V_{Ware}=0. Turbulent pinch $V_{turb} \rightarrow$ density peaking?

Density profile depends on safety factor and temperature

Two additive contributions:

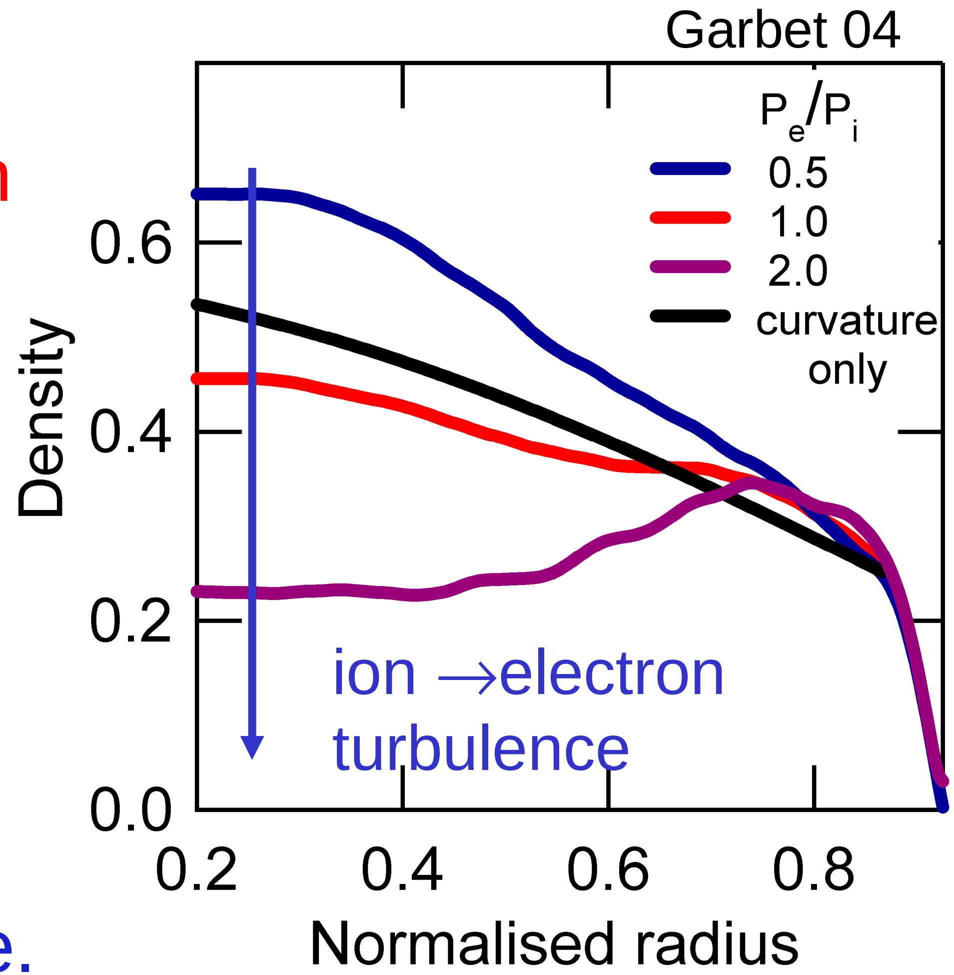
• Curvature pinch, depends on magnetic shear

$$\frac{V}{D} \propto s \qquad s = \frac{r}{q}\frac{dq}{dr}$$

• Thermo-diffusion

$$\frac{V}{D} \propto \frac{\nabla T_e}{T_e}$$

changes sign when moving from electron to ion turbulence.



Momentum transport and spontaneous spin-up

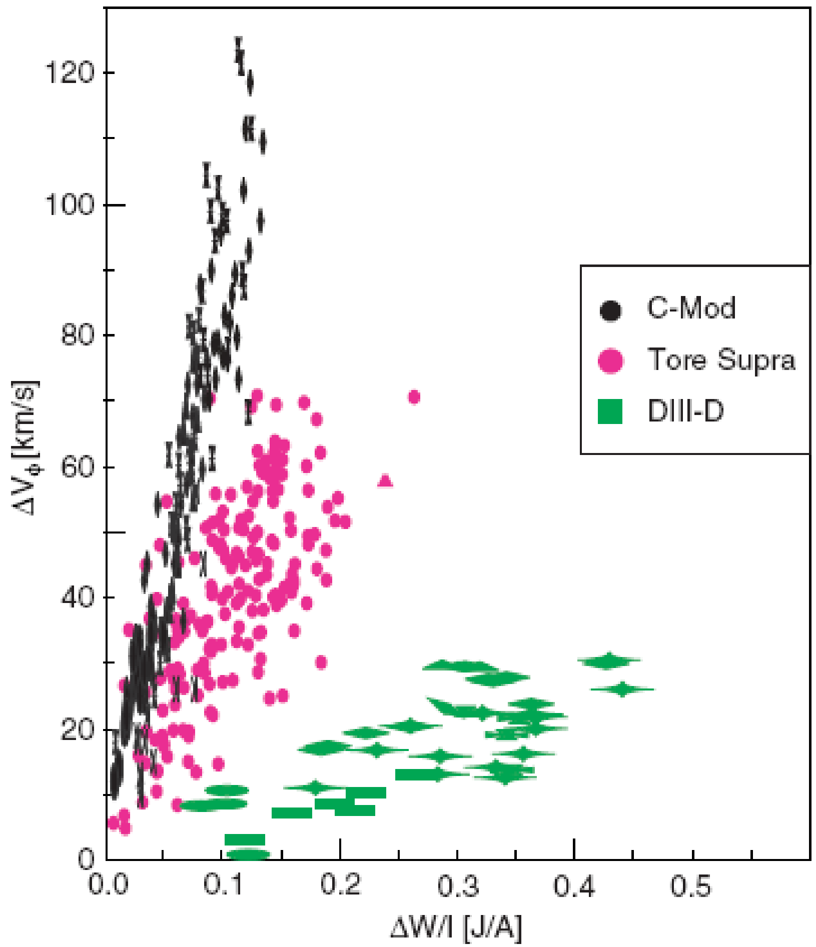
- A puzzling observation on Alcator C-mod, JET, TS, DIII-D: toroidal rotation without external torque
- Structure of momentum radial flux Diamond 07

$$\Gamma_{\Omega} = -D\frac{d\Omega_{\varphi}}{dr} + V\Omega_{\varphi} + S$$

pinch residual stress

- Still an open issue Hahm 06, Gurcan 06, Peeters 07, Waltz 07

Rice 07



Part IV
Building a Transport Model

- Integrated modelling: important for ITER - preparation of scenarios, safe operation, coherence of data, designing control algorithms
- Reduced models for turbulent transport using the Mixing Length Estimate.
- Combining similarity and mixing-length estimate.
- Critical gradient models.

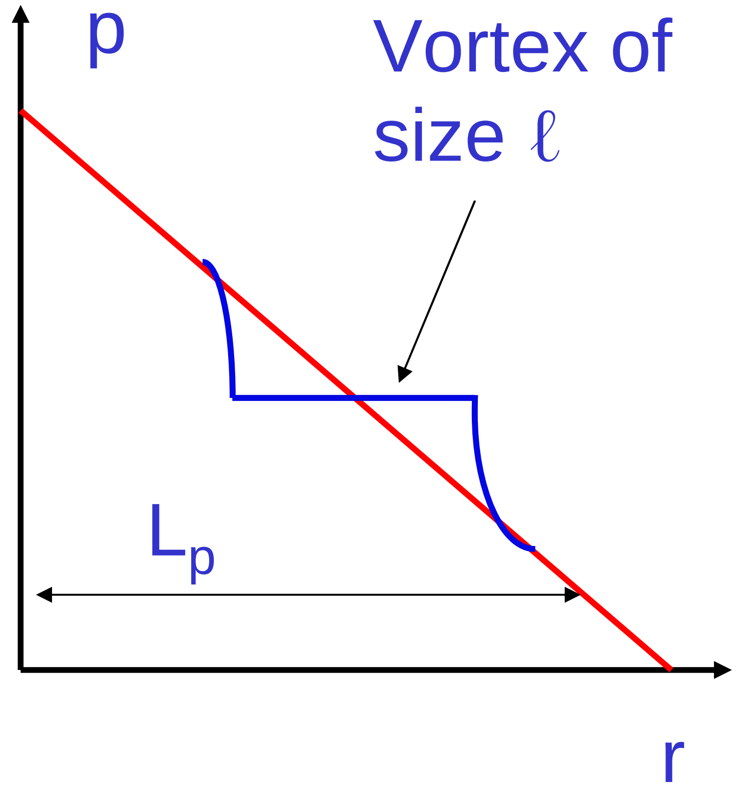
Mixing-length estimate : level of fluctuations

- Mixing of the pressure profile by vortex of size ℓ

$$\frac{\delta p}{p} \approx \frac{\ell}{L_p}$$

- With a bit of cooking …

$$\frac{e\delta\phi}{T} \approx \frac{\delta p}{p} \approx \frac{\gamma}{\omega_r}\frac{\ell}{L_p}$$



Mixing-length estimate : diffusion

- Quasi-linear diffusion Vedenov 61, Drummond 63, Horton 83

$$D = \sum_{\ell} |v_{E\ell}|^2 \tau_{c\ell}$$

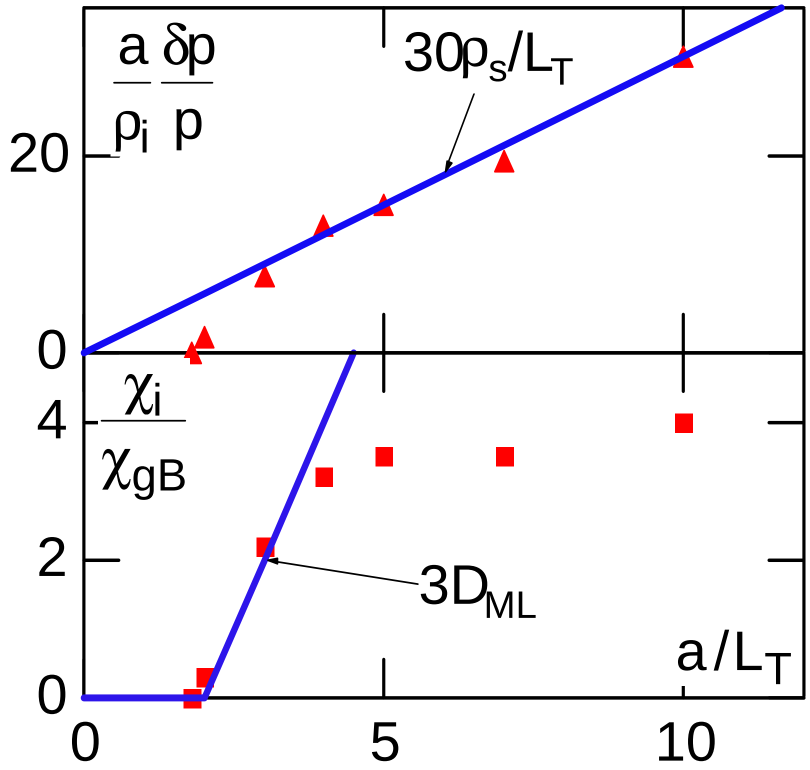
- Combining with mixing-length estimate

$$D \approx \gamma_{max} L_c^2$$

- Basis of most transport models: GLF23, Weiland, CDBM…
- Firmer basis from more refined statistical theories

Diamond 91, Krommes 97, Itoh 99.

Waltz 1994



Critical Gradient Models

- Rules for correlation length and time :

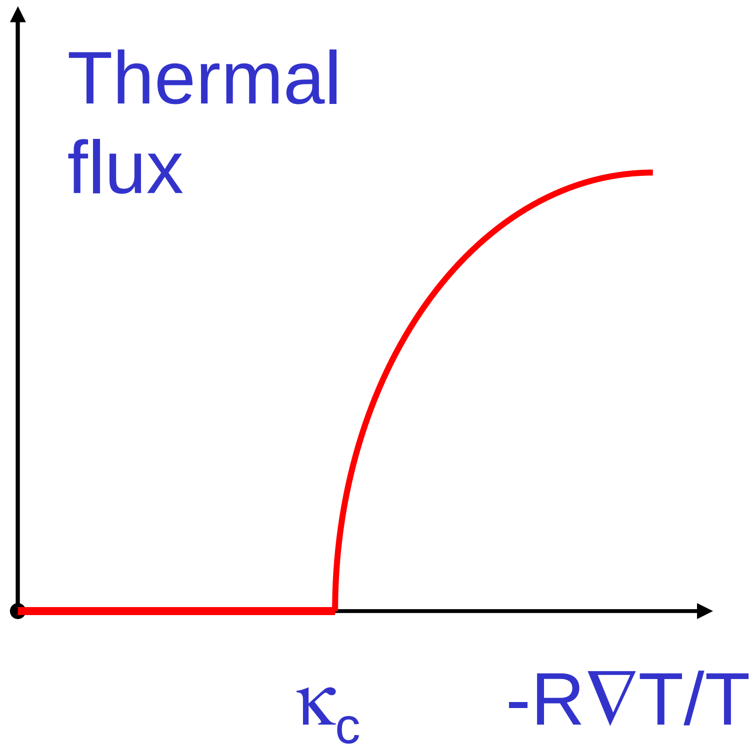
$$L_c \equiv \rho_s \qquad \gamma \equiv \frac{c_s}{R}\left(\left|\frac{RdT}{Tdr}\right| - \kappa_c\right)$$

- Mixing length estimate :

$$\chi = \chi_s \frac{T}{eB}\rho_*\left(\left|\frac{RdT}{Tdr}\right| - \kappa_c\right)$$

stiffness threshold

- Typical behavior of more complex models: Weiland, GLF23, CDBM, …



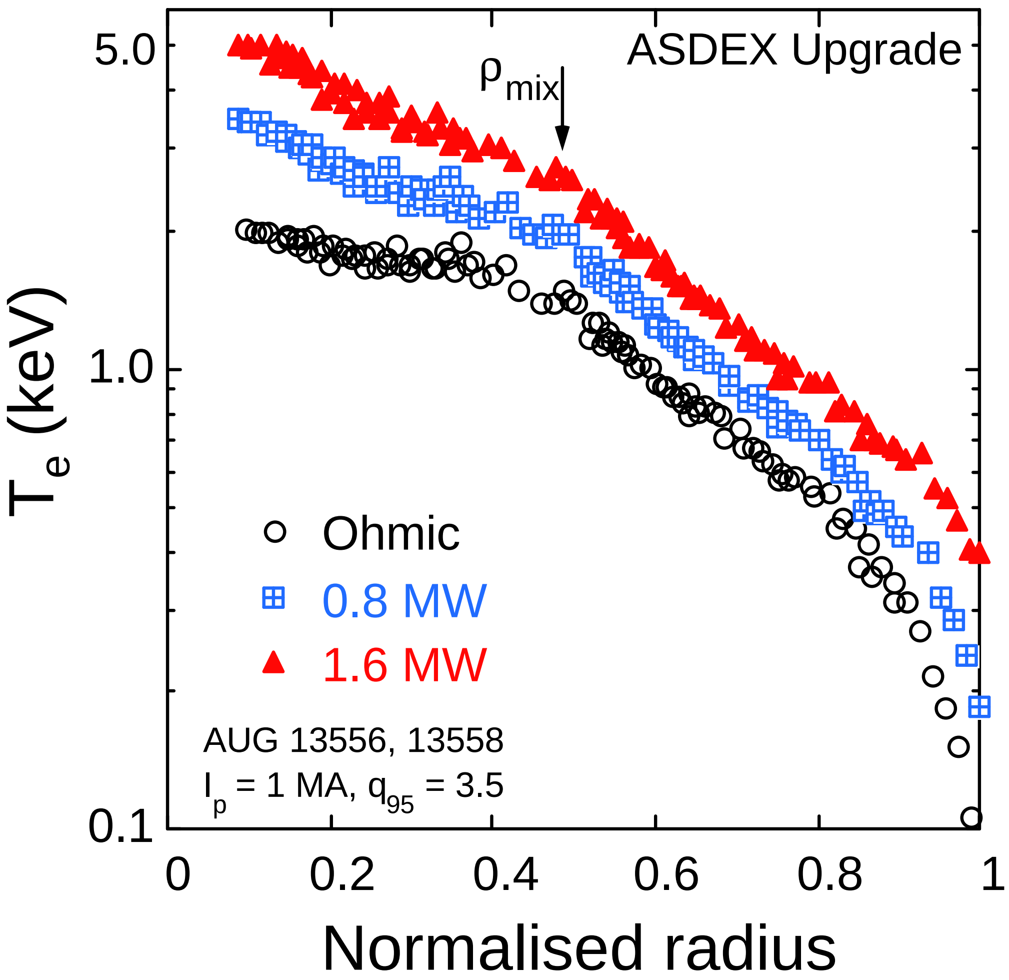
A useful, but controversial, concept : marginal stability

- Marginally stable profile

$$T = T_a e^{\kappa_c \frac{a-r}{R}}$$

- Stiffness: tendency of profiles to stay close to marginal stability.
- Central temperature is improved if

- threshold κ_c is larger
- edge pedestal T_a is higher.



Development of reduced models: present status

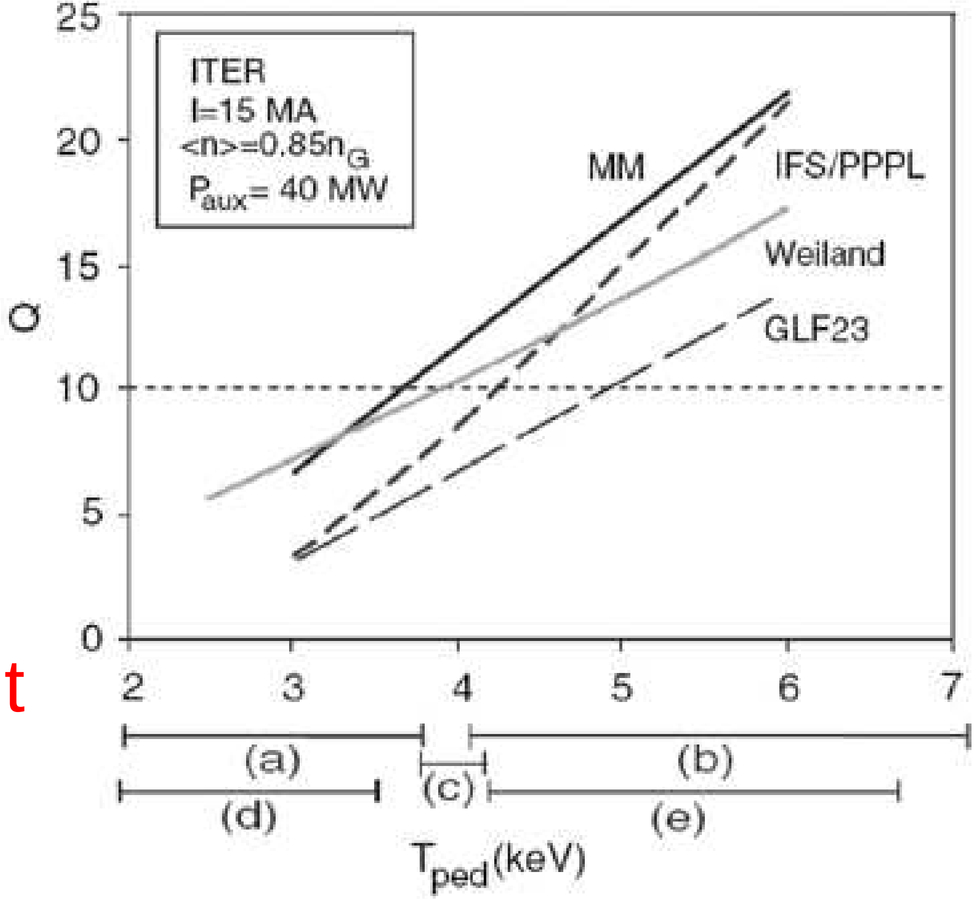
- Encouraging results

see lecture by Pr Fukuyama.

- However, still some uncertainty on the prediction of ITER performances.

- Requires an improvement on transport models.

Iter Physics Basis



Part V - Beyond the Mixing Length Estimate

- Tendency for producing large scale structures: inverse cascade.
- Large scale transport events: avalanches and streamers: breaks locality and scaling of the correlation length, some link with turbulence spreading.
- Fluctuations of the poloidal flow: Zonal Flows, Geodesic Acoustic Modes. Reduce anomalous transport. Introduce non locality in k space.
- Sources of intermittency.

Large Scale Transport Events

- Events that take place over distances larger than a correlation length

- Identified as

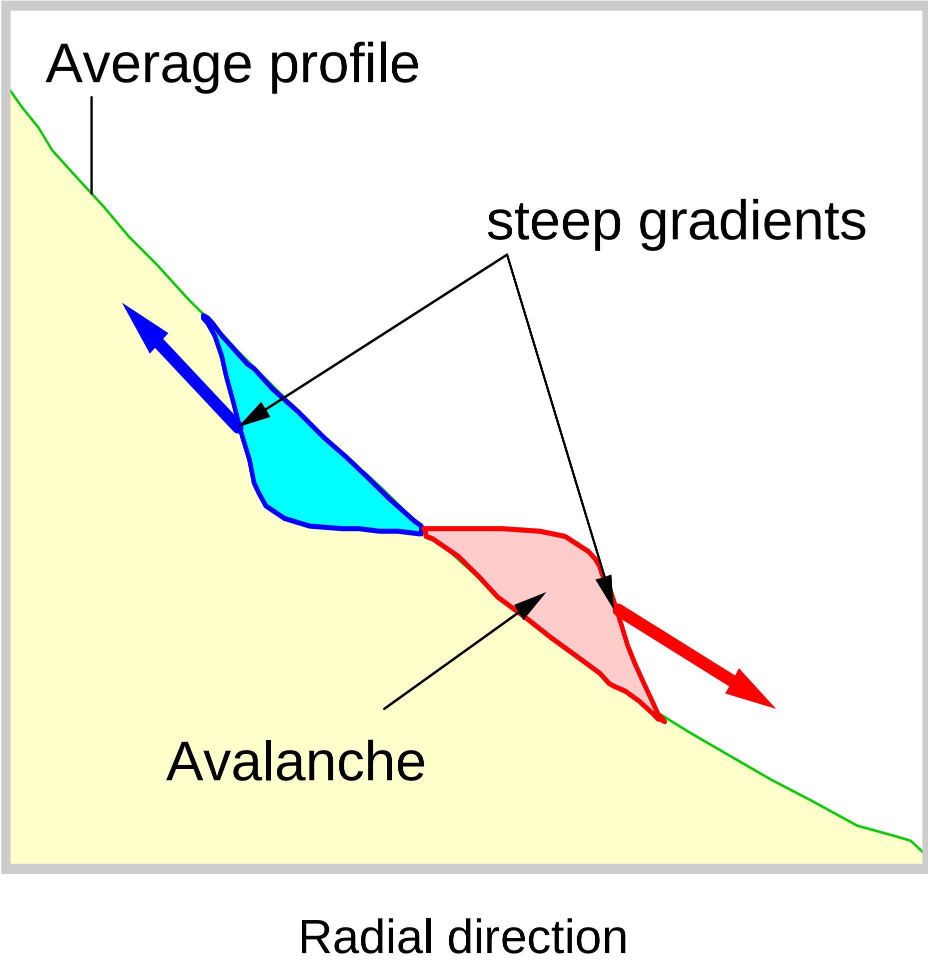
  - avalanches

  - streamers

- May lead to enhanced transport and/or non local effects.

Avalanches

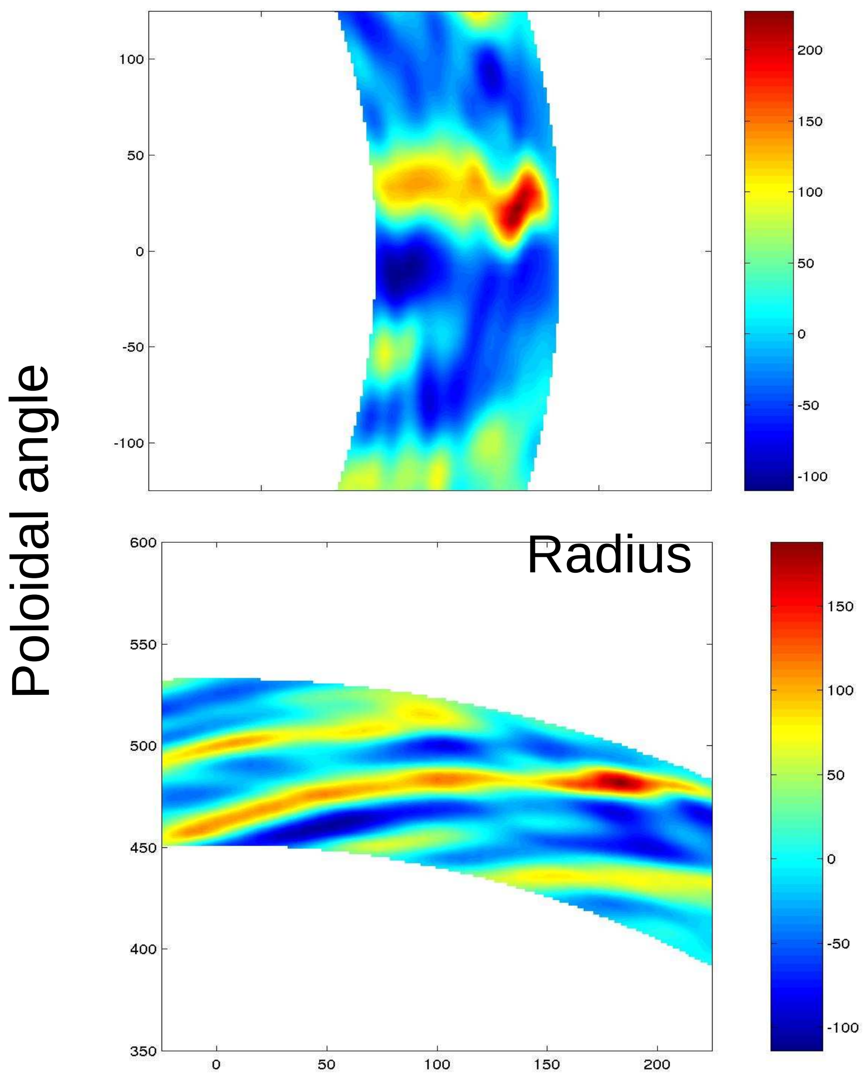
- Profile relaxations at all scales. Diamond&Hahm 95.
- Domino effect.
- Propagate at a fraction of the sound speed.
- Clear link with turbulence spreading. Garbet 94, Hahm 04



Streamers

- Convective cells elongated in the radial direction, aligned along the magnetic field. Beyer 00, Champeaux 00.
- Boost the radial transport if the ExB velocity is large enough $\rightarrow$ controversial. Jenko 00, Labit 03, Idomura 06, Lin 05, Candy 08.

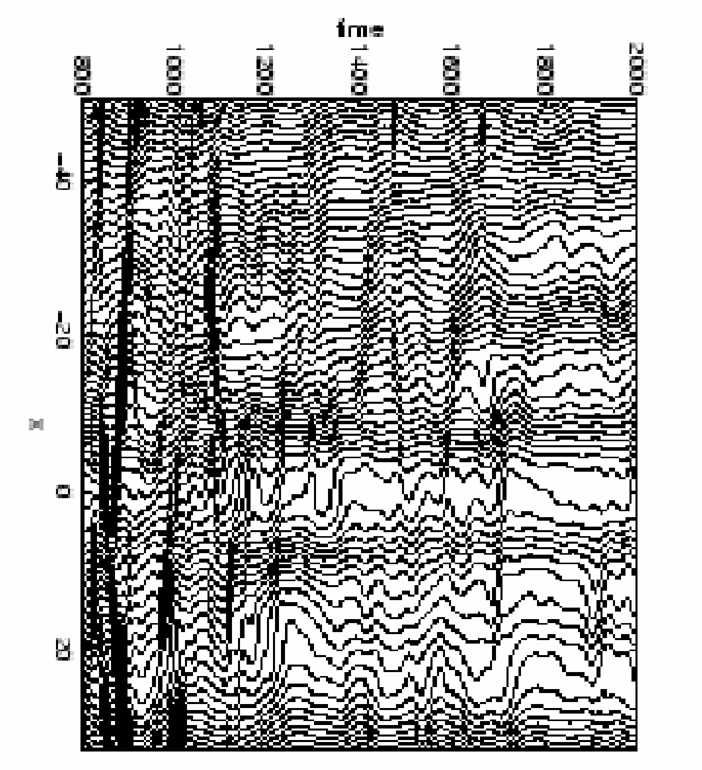
RBM simulations



Do avalanches and streamers really exist?

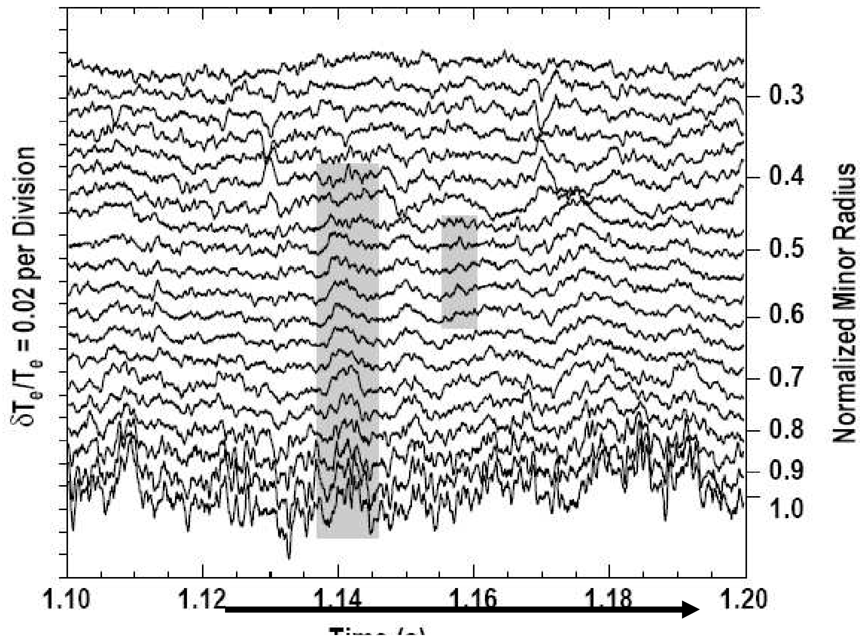
- No direct observation.
- Some hint from fast evolution of temperature profiles.

x



Beyer 00

Politzer 00



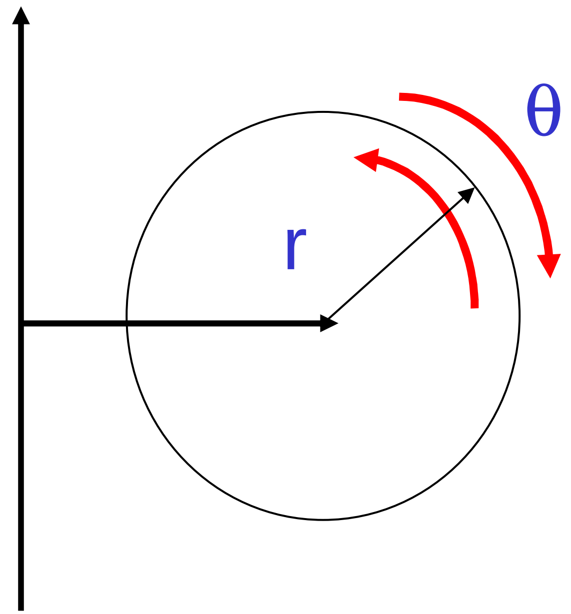
T(x,t)

Time (s)

Zonal flows

Diamond, Itoh, Itoh & Hahm 05

- Fluctuations of the poloidal velocity
- Generated by turbulence via Reynolds stress
- Damping is weak Rosenbluth & Hinton 98

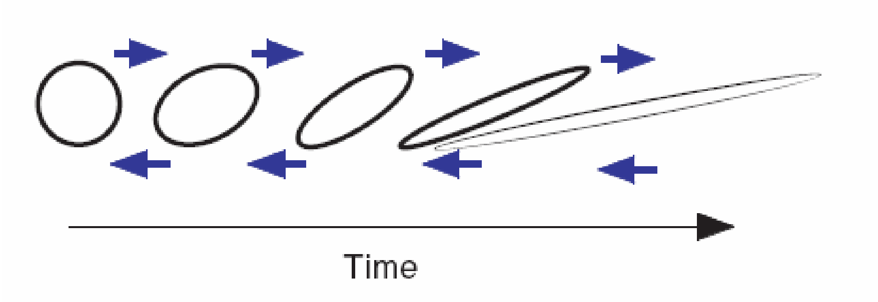


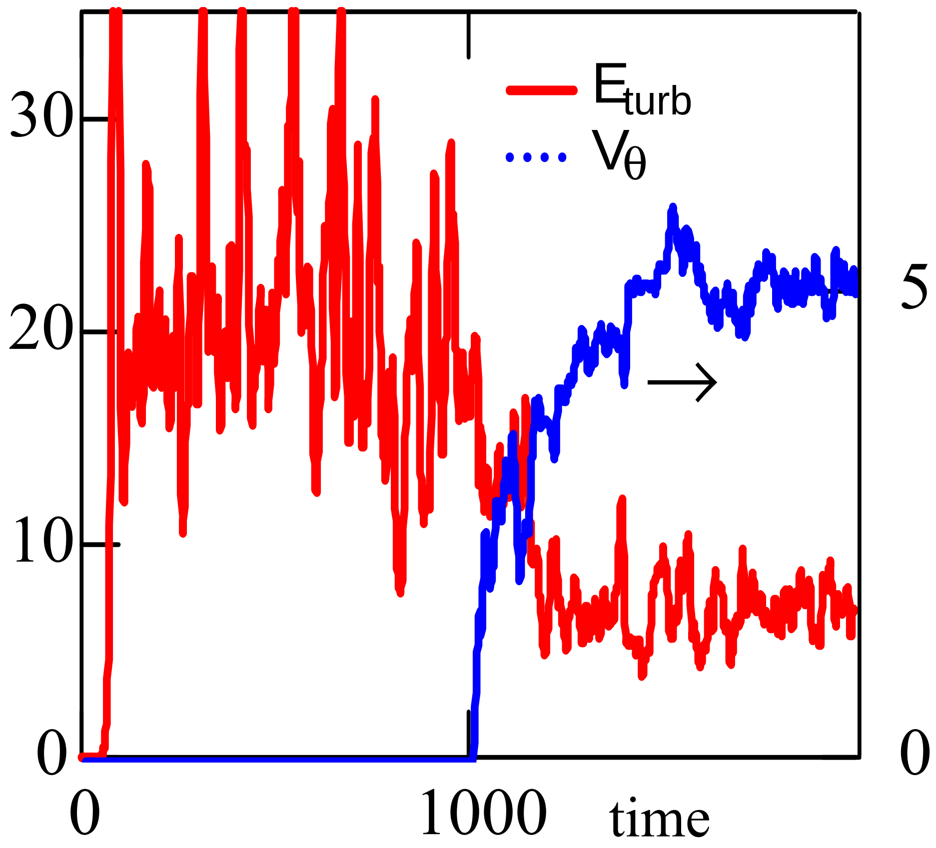
$$\partial_t V_\theta' = -\nabla_r \cdot \underbrace{\left\langle v_{Er} v_{E\theta} \right\rangle}_{} - \nu V_\theta'$$

Turbulent amplification ~ $|\phi|^2 V_\theta'$

Zonal Flows (cont.)

- Strong feed-back on turbulence: shearing of vortices.
- Clearly seen in all turbulence simulations.
- Leads to a self-organized state





Geodesic Acoustic Modes

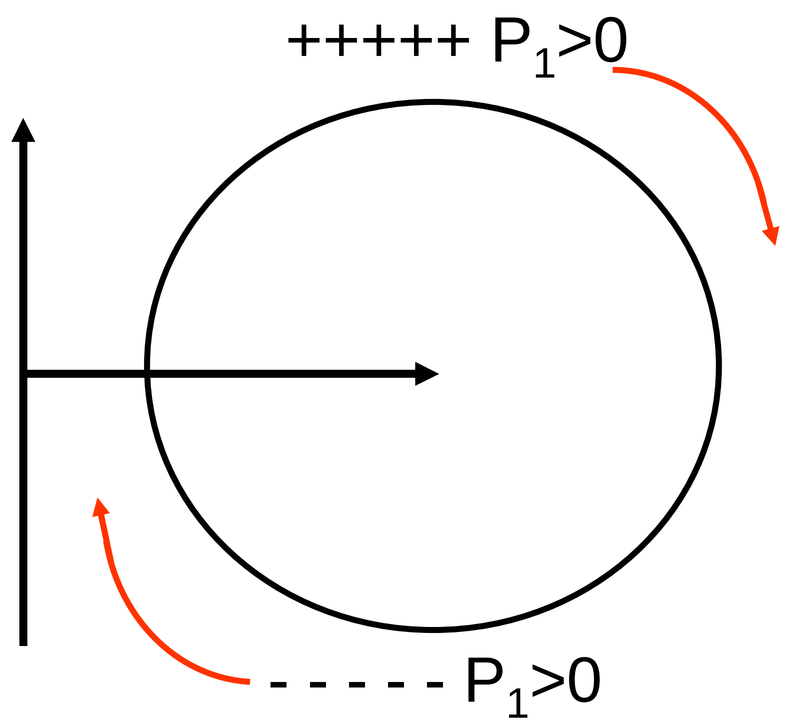
- n=0, m=0 mode coupled to sidebands m= ±1, m±2, …

Hallatschek 01, cluster PPCF 06

- GAM frequency

$$\omega^2 = \left(1 + \frac{1}{2q^2}\right) \frac{2\Gamma T_i + T_e}{m_i} \frac{1}{R^2}$$

- Turbulence self-regulation, however shear effect less efficient than zonal flows.



$$V_0 = -\frac{\partial_r \phi_0}{B}$$

Impact on Transport Models

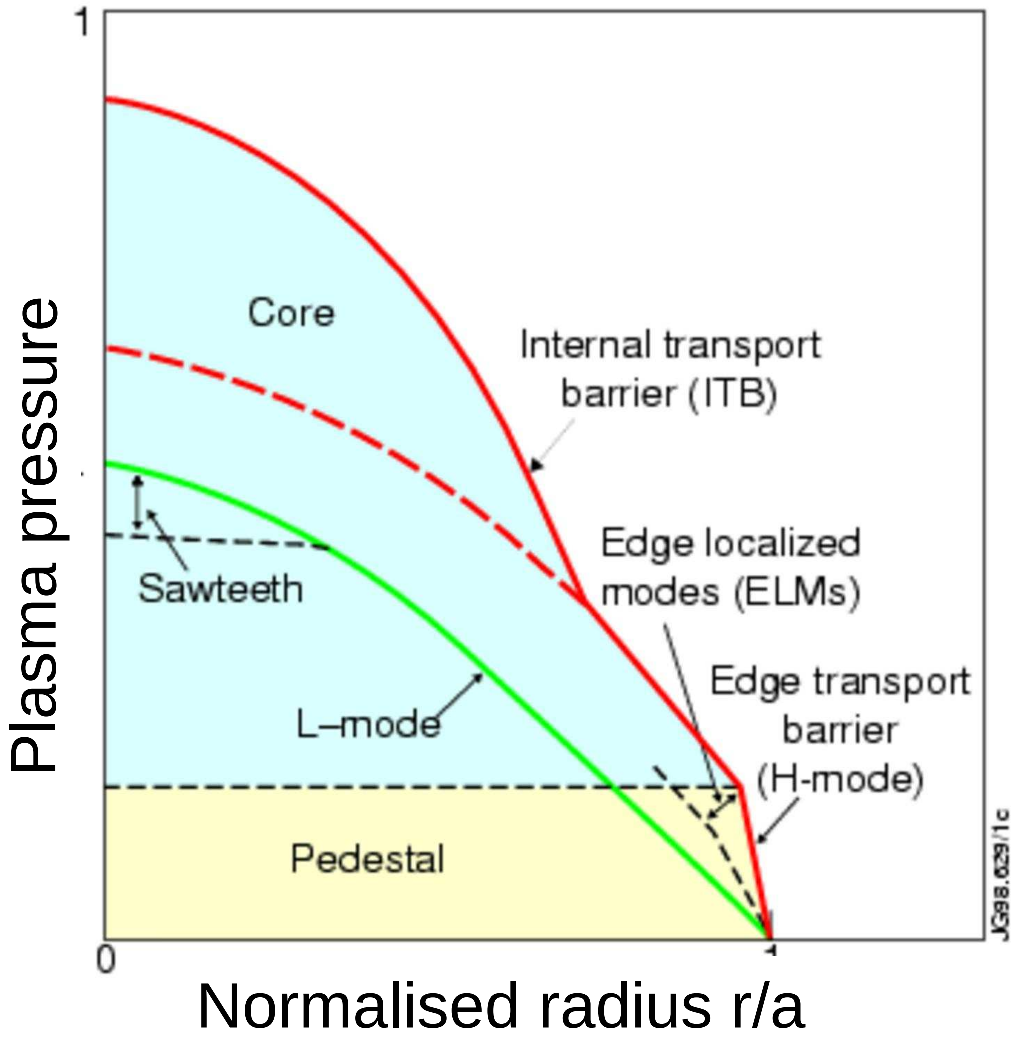
- Mixing-length estimate can be modified to account for Zonal Flows (GLF23, Weiland, …): some cooking !
- Statistical theory accounting for all these beasties still to be fully developed ...
- Why not direct simulations of turbulence, as for weather forecast?

Part VI
Improved confinement

- Shear flow
- Negative magnetic shear
- Transport barriers
- Consequences

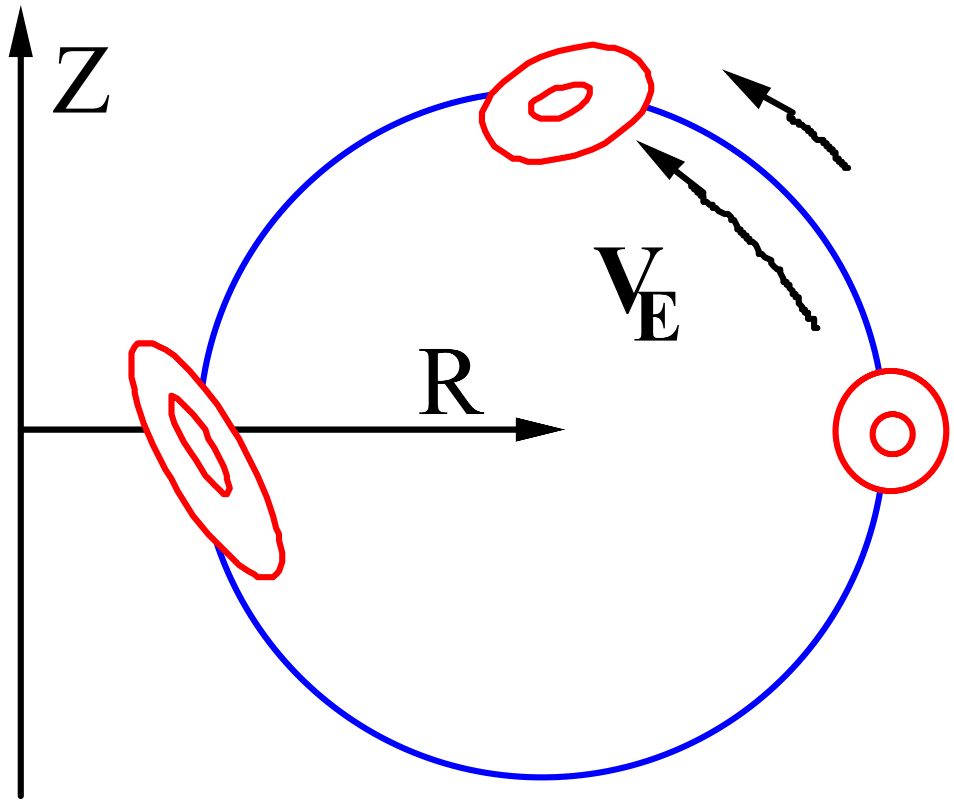
Several “regimes” in a tokamak plasma

- L-mode: basic plasma, turbulence everywhere.
- H-mode: low turbulent transport in the edge, formation of a pedestal.
- Internal Transport Barrier: low turbulent transport in the core, steep profiles.



Several mechanisms may lead to improved confinement

- Flow shear: same effect as Zonal Flows
- Magnetic shear
- T_e/T_i, Z_{eff}, density gradient, fast particles… : not generic

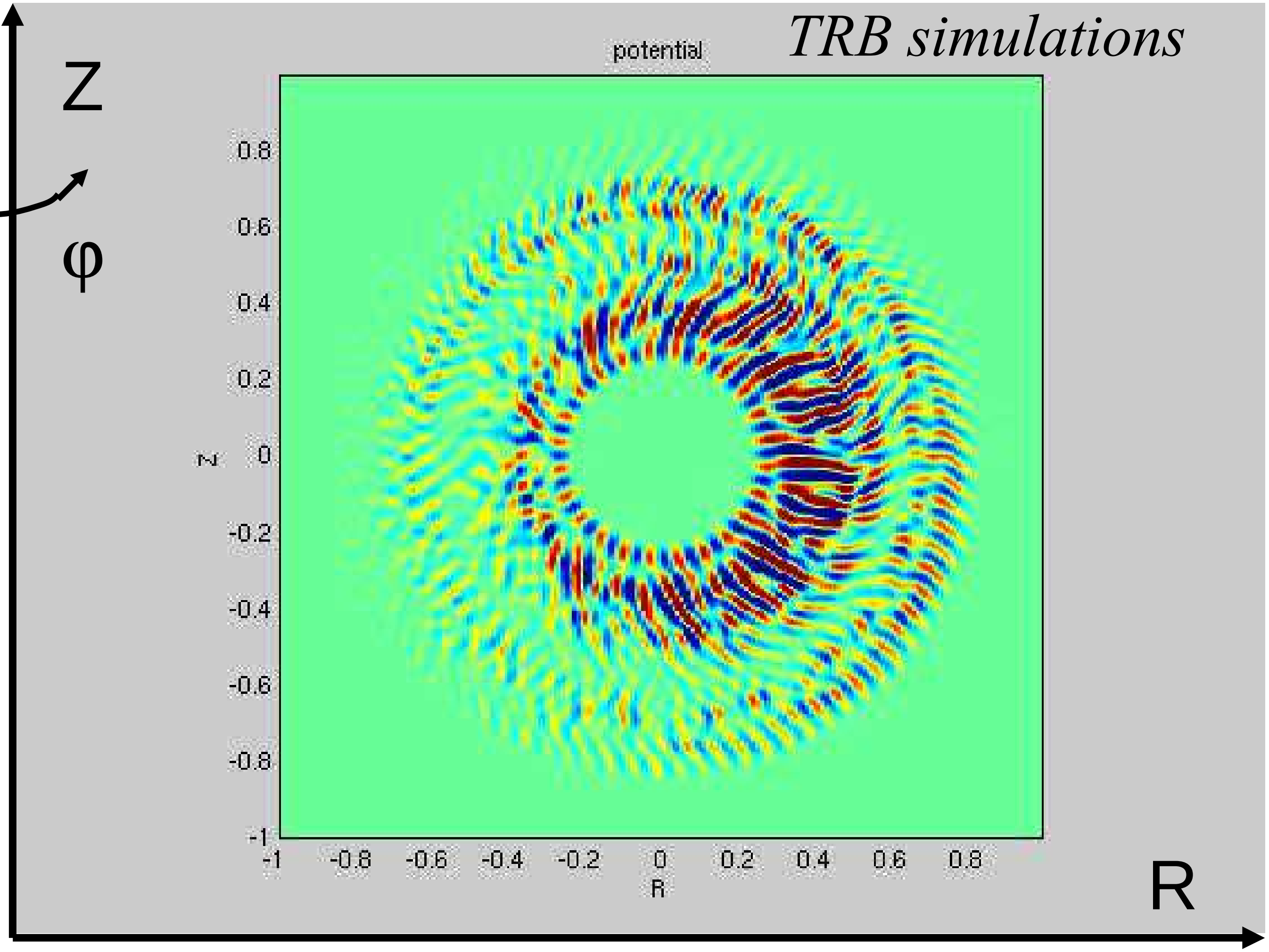


Shear flow is stabilizing

- E×B velocity shear tears apart large scale vortices

- Approximate criterion for stabilization

$$\gamma_E = \frac{dV_E}{dr} > \gamma_{lin}$$



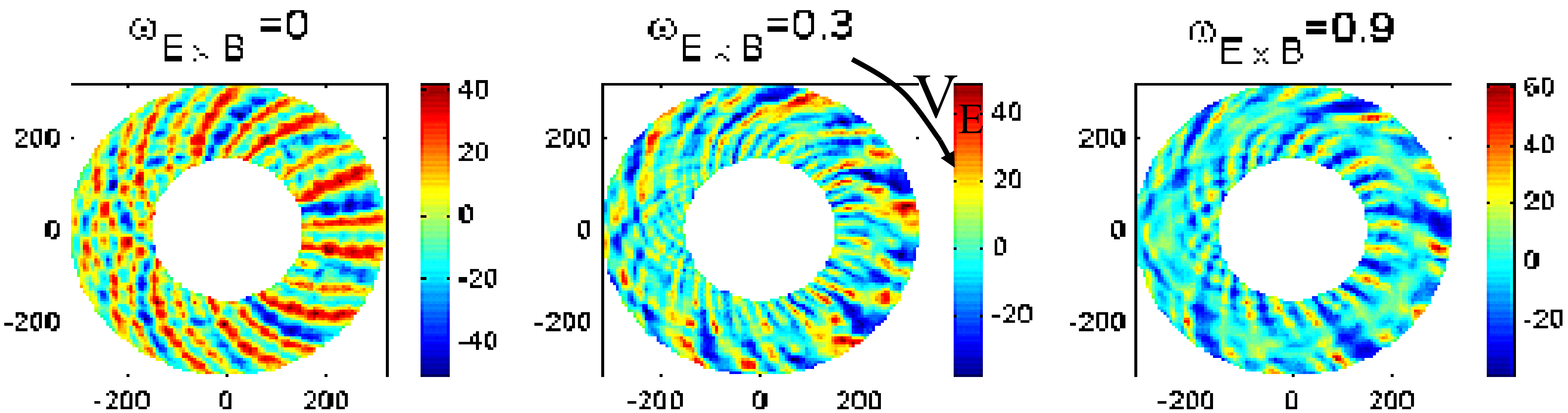
Contour lines of electric potential.

Flow shear stabilisation

- Shear rate $V_E^{'} = \frac{dV_E}{dr}$
- Criterions for stabilization

$$\left(Dk_\theta^2 V_E^{'2}\right)^{1/3} > \tau_c^{-1} \qquad\qquad V_E^{'} > \gamma_{lin}$$

Biglari-Diamond-Terry 90 Waltz 94



Figarella 03

Controlling the flow

- Force balance equation

$$E_r = \frac{T_i dn_i}{e_i n_i dr} + (1 - k_{neo}) \frac{dT_i}{e_i dr} + V_{Ti} B_p$$

Fuelling   Heating   Toroidal momentum

$\rightarrow$ power threshold!

- Flow generation

$$\partial_t V_\theta = -\nabla_r \langle \tilde{V}_{Er} \tilde{V}_{E\theta} \rangle - \nu_{neo} \left(V_\theta - V_{eq} \right)$$

Transport reduction due to shear flow

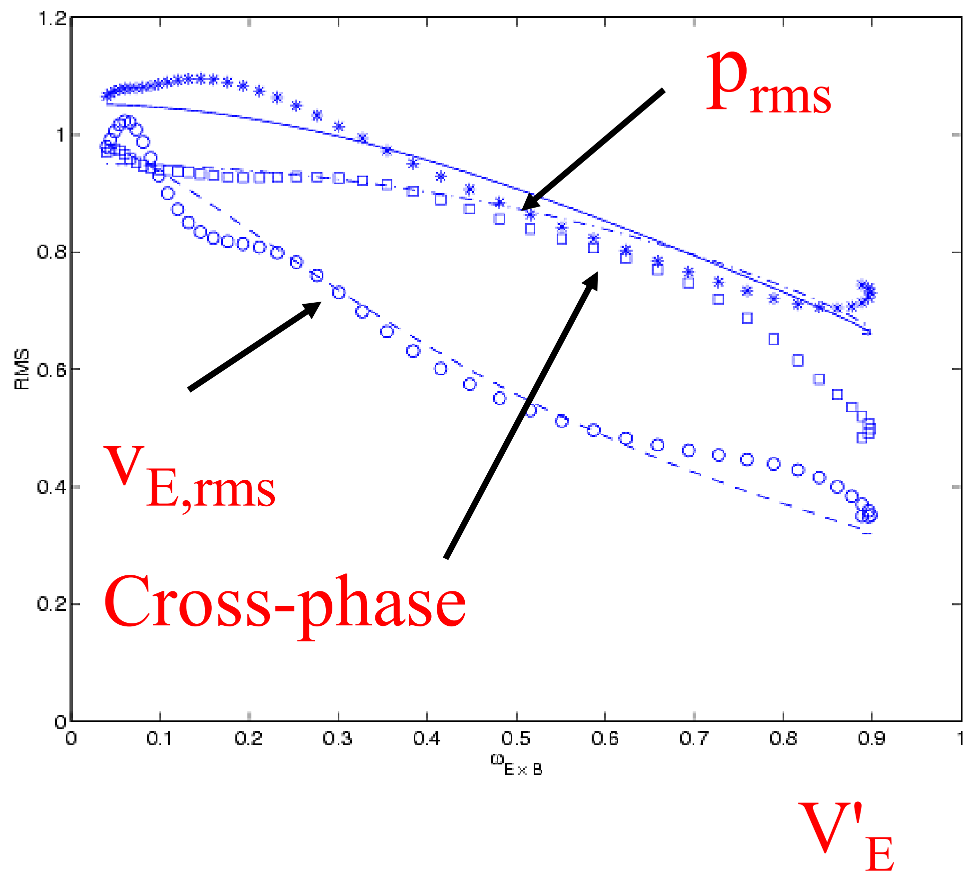
- Flux reduction factor factor $F(\gamma_E)$

$$F = \frac{1}{1 + \left(\frac{V_E'}{\gamma_{lin}}\right)^2}$$

or

$$F = 1 - \frac{V_E'}{\gamma_{lin}}$$

Figarella 03

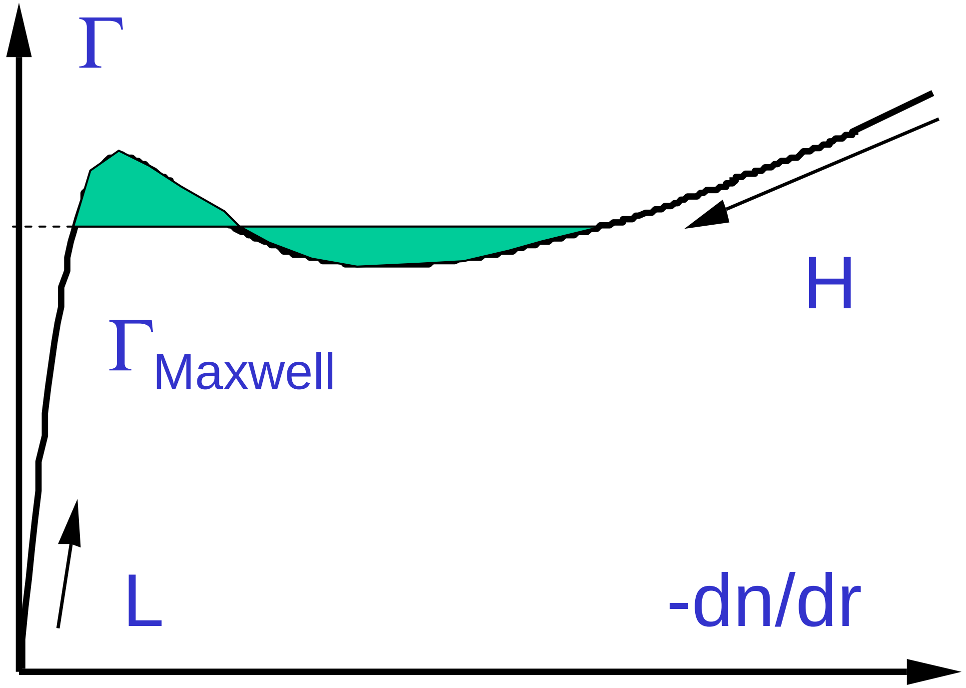


A simple model for a bifurcation towards a transport barrier

- Particle flux with ExB shear

$$\Gamma = -D \frac{1}{1 + C\left(\frac{dn}{dr}\right)^4} \frac{dn}{dr}$$

- Transition to improved confinement occurs above a critical threshold in flux.



Hinton 92, Itoh 02, Diamond 07

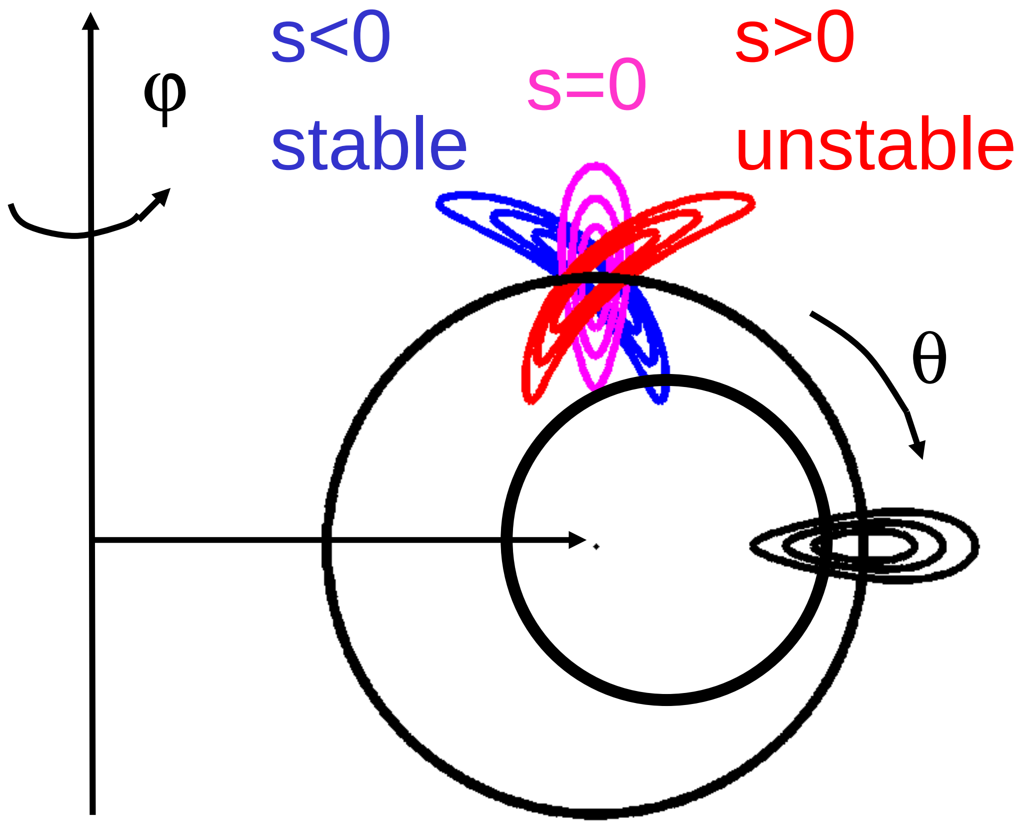
Negative magnetic shear is stabilising

- Magnetic shear :

$$s = \frac{r}{q}\frac{dq}{dr}$$

- s<0 : favourable average of interchange drive $(v_E \cdot \nabla B)(v_E \cdot \nabla p)$ along field lines.
- Enhanced by geometry effect.

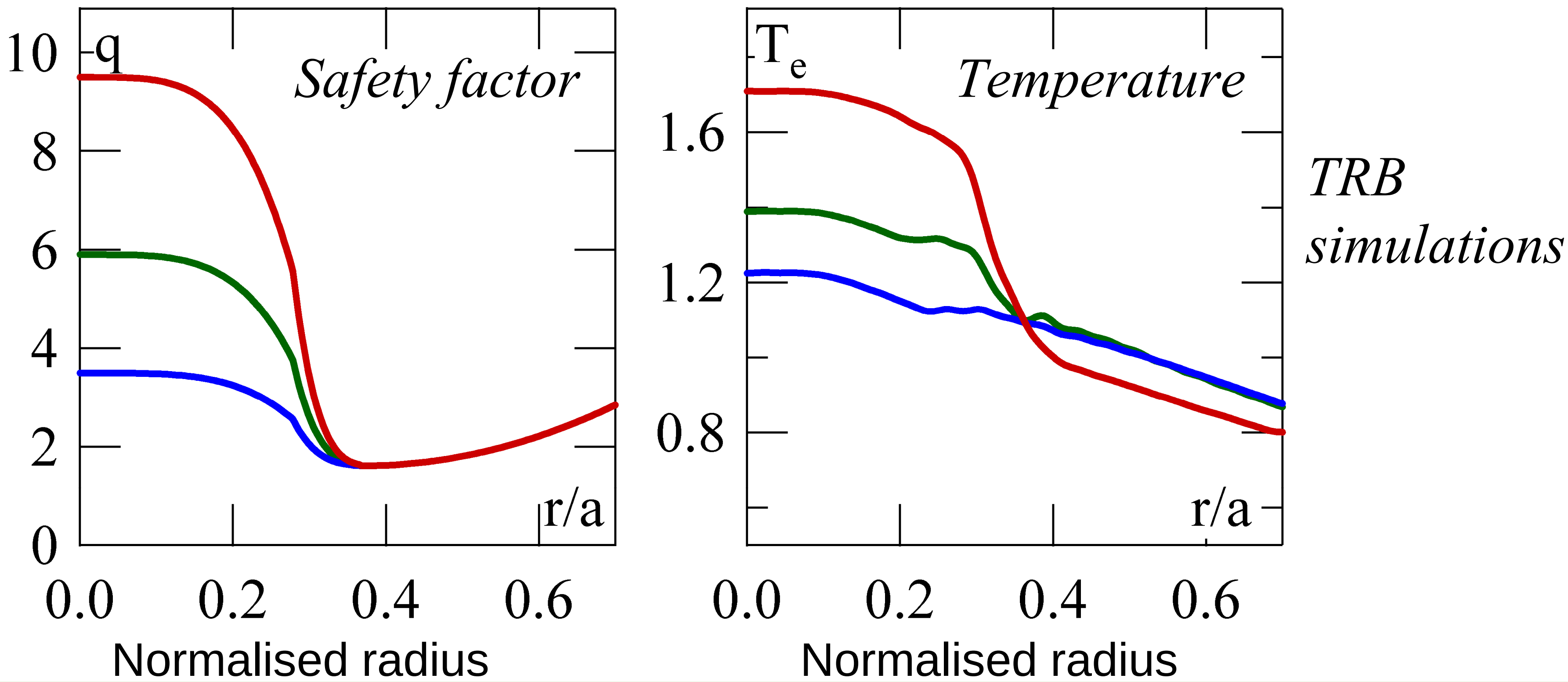
B.B.Kadomtsev, J.Connor, M.Beer, J.Drake, R.Waltz, A.Dimits, C.Bourdelle…



Vortex distorsion

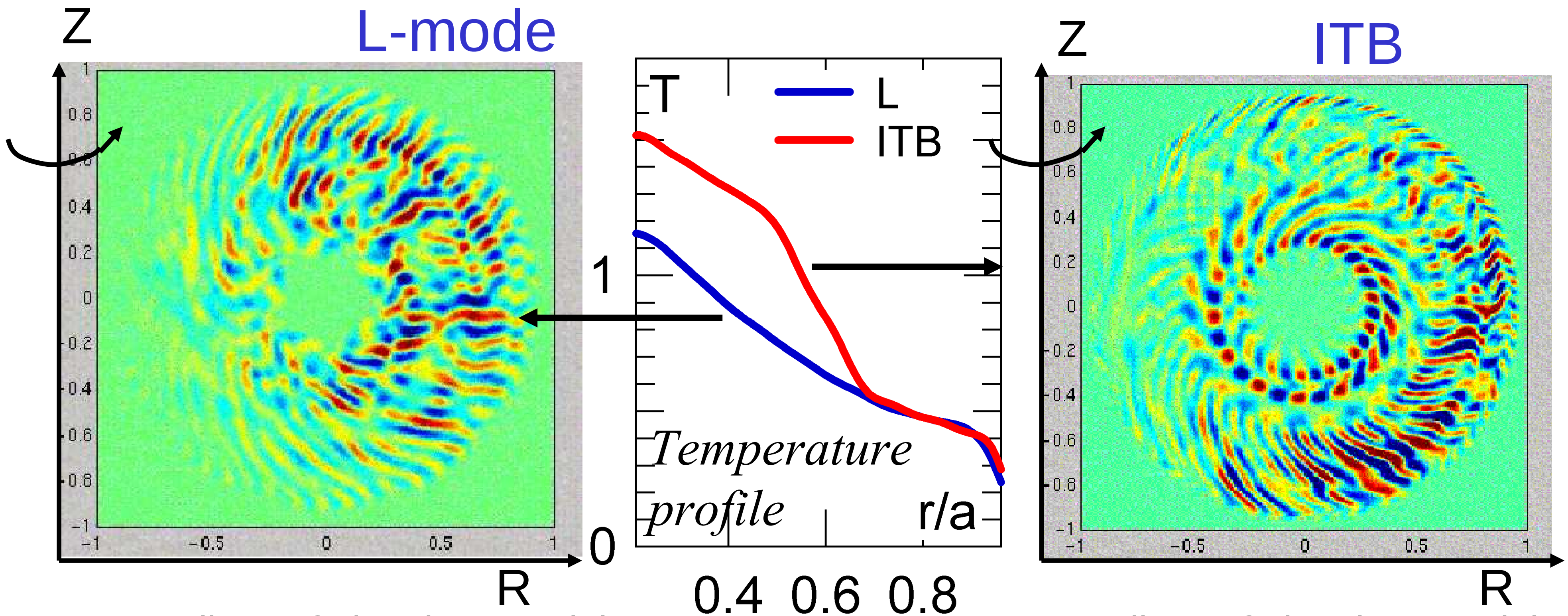
Negative magnetic shear is a robust effect

- Turbulence simulations : stabilization for s<-0.5
- Some agreement with electron transport barriers in JET



Internal Transport Barriers

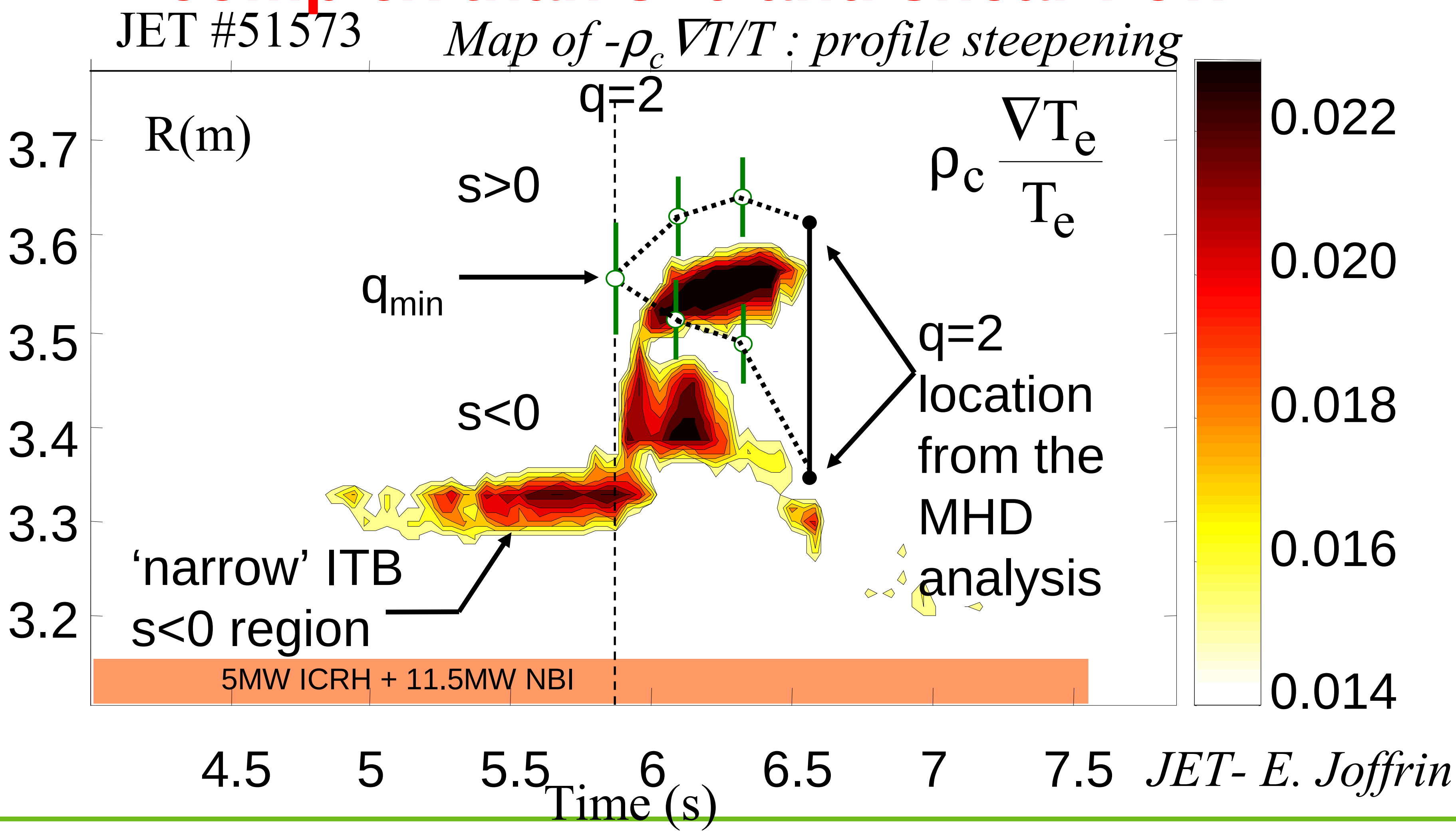
- Transport barriers are layers of plasma where turbulent transport is quenched.
- Requires a minimum amount of power → triggering?



Contour lines of electric potential.

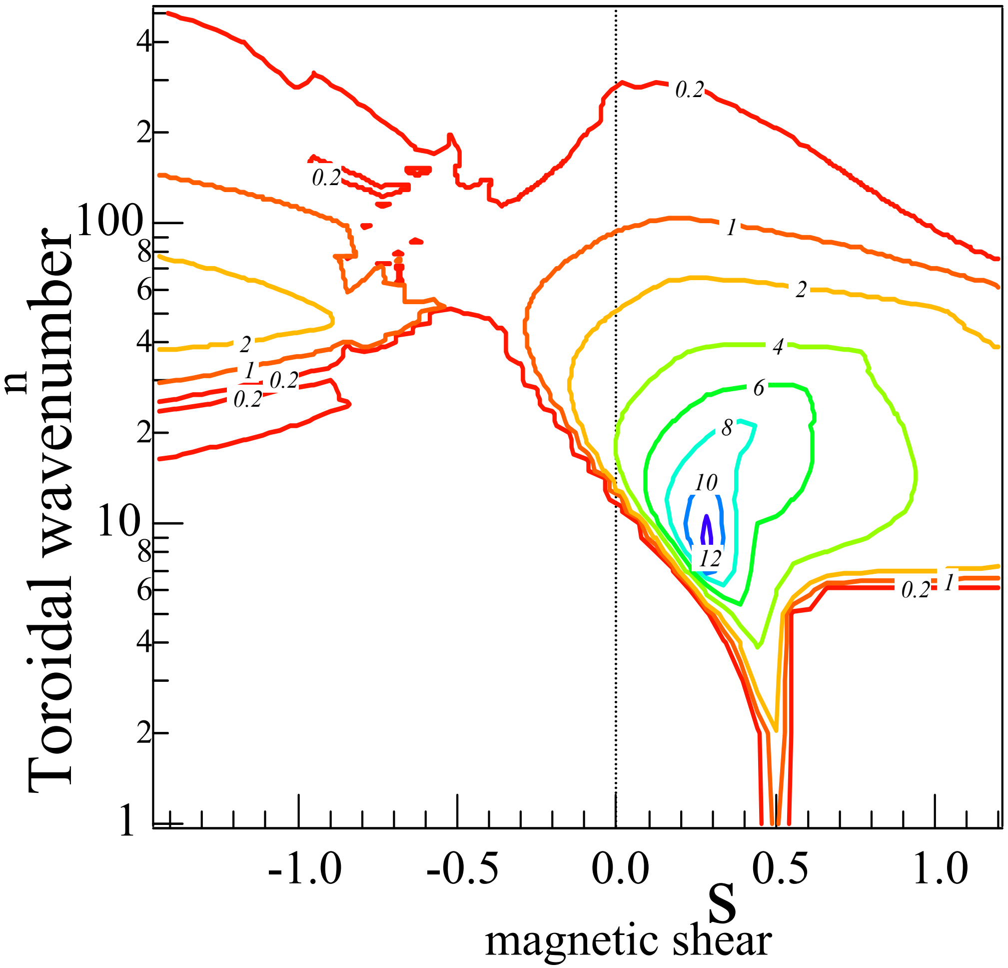
Contour lines of electric potential.

Dynamics of transport barriers is more complex than s<0 and shear flow



Why a special role of s=0 and rational surfaces?

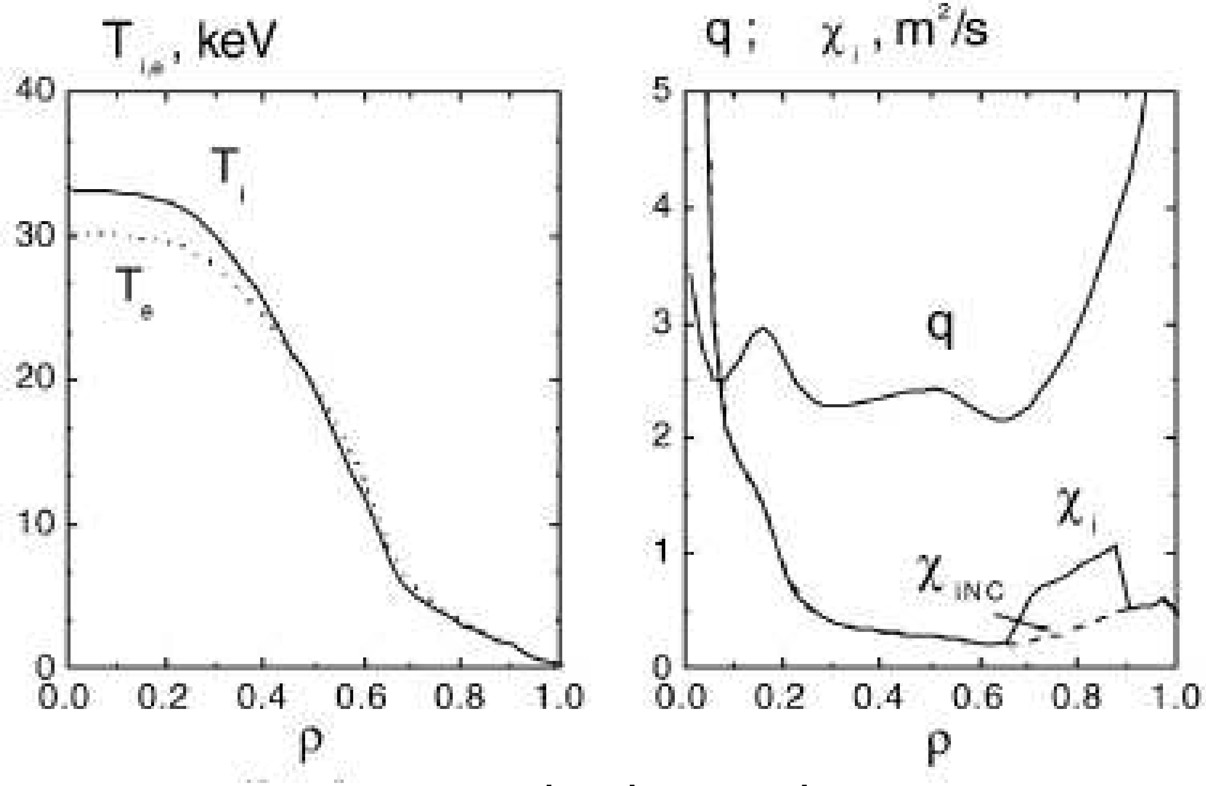
- At negative shear, slab ITG still unstable → some kind of optimum for s=rdq/qdr=0.
- Special role of low order rational surfaces:
- density of resonant surfaces Romanelli 93, Kishimoto 99, Garbet 01
- MHD mode Joffrin 04
- Zonal flows Waltz 06
- convective cell Diamond 06



Consequences for ITER: advanced scenarios

Advanced scenarios where the plasma current is non inductively generated are foreseen in a second phase.

- The objective is to reach a steady-state regime : needs a large fraction of bootstrap current.
- Requires an ITB or some global improvement of the confinement.



Iter Physics Basis

Conclusions I

- Huge progress in the understanding of turbulent transport, thanks to theory, turbulence simulations and increasingly refined measurements.

- Some hotly debated issues though: dimensionless scaling laws, electron heat transport, particle and momentum transport.

- Present computational resources do not allow a full scale turbulence simulation for ITER.

Conclusions II

- Reduced transport models are efficient ways of testing theories, analysing experiments, and predicting performances in ITER. Still the accuracy of reduced transport models is not better than 20%.
- Due to the complex dynamics of turbulence: structure formation, intermittency, etc,...
- Improved models on the basis of a better statistical theory (to be done) or direct use of simulations of turbulence?

Conclusions III

- Generic mechanisms to control turbulence $\rightarrow$ improved confinement. Crucial for ITER.
- Turbulence simulations are good tools to test the validity of various theoretical ideas.
- Still many issues remain unresolved. At the moment, no full ab-initio simulations of L-H transition.
- Long pulse plasmas in ITER with improved confinement will be a challenge.